15/ENG02/095

Ogoegu Patrick Ofili

Civil Engineering

ENG 881

① $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Solve for LHS = 0 (C.F)

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2) = 0$

$m = -1 \quad m = 2$

$y = Ae^{m_1x} + Be^{m_2x}$

$y = Ae^{-x} + Be^{2x}$

Solve for RHS (PI)

$f(x) = 8$

$y = c$

$y' = 0 \quad y'' = 0$

Substituting

$0 - 0 - 2c = 8$

$-2c = 8$

$c = -4 \quad \therefore PI = -4$

GS = CF + PI

$y = Ae^{-x} + Be^{2x} - 4$

② $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

LHS (CF) = 0

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm 2$

$y = A\cosh nx + B\sinh nx$

$= A\cosh 2x + B\sinh 2x$

RHS (PI)

$f(x) = 10e^{3x}$

$y = ce^{3x}$

$y' = 3ce^{3x}$, $y'' = 9ce^{3x}$

Substituting

$9ce^{3x} - 4ce^{3x} = 10e^{3x}$

$9c - 4c = 10$

$5c = 10$

$c = 2$, $PI = 2$

$y = ce^{3x}$

$y = 2e^{3x}$

$GS = CF + PI$

$= A\cosh 2x + B\sinh 2x + 2e^{3x}$

5) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

LHS (CF) = 0

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1$, $m = -1$

$m_1 = m_2 = m = -1$

$y = e^{mx}(A + Bx)$

$y = e^{-x}(A + Bx)$

RHS (PI)

$f(x) = e^{-2x}$

$y = ce^{-2x}$

$\frac{dy}{dx} = -2ce^{-2x}$

$\frac{d^2y}{dx^2} = 4ce^{-2x}$

Substituting

$4ce^{-2x} - 4ce^{-2x} + Ce^{-2x} = e^{-2x}$

$4c - 4c + c = 1$

$c = 1$

$y = Ce^{-2x}$

$y = e^{-2x}$

$GS = CF + PI$

$= e^{-x}(A + Bx) + e^{-2x}$

4) $\dfrac{d^2y}{dx^2} + 25y = 5x^2 + x$

LHS (CF) = 0

$m^2 + 25 = 0$

$m^2 = -25$

$m = \sqrt{-25}$

$m = \pm j\sqrt{25}$

$m = \pm j5$

$y = A\cos nx + B\sin nx$

$y = A\cos 5x + B\sin 5x$

RHS

$f(x) = 5x^2 + x$

$y = Cx^2 + Dx + E$

$y' = 2Cx + D$

$y'' = 2C$

Substituting

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 + 25Dx + 2C + 25E = 5x^2 + x$

x^2: $25Cx^2 = 5x^2$

$25C = 5$

$C = \dfrac{\cancel{5}}{\cancel{25}} = \dfrac{1}{5}$

x: $25Dx = x$

$25D = 1$

$D = \dfrac{1}{25}$

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$2/5 = -25E$

$\dfrac{2}{5} \times \dfrac{1}{-25} = E$

$E = -2/25$

$y = Cx + Dx + E$

$= \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$= \frac{1}{125}(25x^2 + 5x - 2)$

$GS = CF + PI$

$= A\cos x + B\sin x + \frac{1}{125}(25x^2 + 5x - 2)$

6)

5) $\frac{dy^2}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

LHS (CF) = 0

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1) = 0$

$m_1 = m_2 = m = 1$

$y = e^{mx}(A + Bx)$

$y = e^{x}(A + Bx)$

RHS

$f(x) = 4\sin x$

$y = C\cos x + D\sin x$

$y' = -C\sin x + D\cos x$

$y'' = -C\cos x - D\sin x$

substitute

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-\cancel{C\cos x} - \cancel{D\sin x} + 2C\sin x - 2D\cos x + \cancel{C\cos x} + \cancel{D\sin x} = 4\sin x$

$2C\sin x - 2D\cos x = 4\sin x$

$2C\sin x = 4\sin x$

$2C = 4$

$C = 2$

$-2D\cos x = 0$

$-2D = 0$

$D = 0$

$y = C\cos x + D\sin x$

$y = 2\cos x + 0$

$GS = CF + PI$

$= e^{x}(A + Bx) + 2\cos x$

6) $d^2y/dx^2 + 4dy/dx + 5y = 2e^{-2x}$

CF : solve LHS = 0

$m^2 + 4m + 5 = 0$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times 5)}}{2 \times 1}$

$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2j}{2} = -2 \pm j$

$y = e^{-2}(A\cos x + B\sin x)$

PI -> $f(x) = 2e^{-2x}$

$y = ce^{-2x}$

$y' = -2ce^{-2x}$, $y'' = 4ce^{-2x}$

substitute into the given equation

$4ce^{-2x} + 4c(-2ce^{-2x}) + 5(ce^{-2x}) = 2e^{-2x}$

$4ce^{-2x} - 8ce^{-2x} + 5ce^{-2x} = 2e^{-2x}$

e^{-2x}: $4c - 8c + 5c = 2$

$c = 2$

$y = 2e^{-2x}$

(GS) $y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$x = 0, y = 1$

$1 = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-2(0)}$

$1 = 1(A + 0) + 2$

$1 = A + 2$

$1 - 2 = A$

$A = -1$

$$y = e^{-2x}(-\cos x + B\sin x) + 2e^{-2x}$$

$$dy/dx = e^{-2x}(\sin x + B\cos x) - 2e^{-2x}(-\cos x + B\sin x) - 4e^{-2x}$$

if $x = 0$ and $dy/dx = -2$

$$-2 = e^{-2(0)}(\sin(0) + B\cos(0)) - 2e^{-2(0)}(-\cos(0) + B\sin(0)) - 4e^{-2(0)}$$

$$-2 = 1(0 + B) - 2(-1 + 0) - 4$$

$$-2 = B + 2 - 4$$

$$B = 0$$

~~ss~~ Particular equation

$$y = e^{-2x}(-\cos x) + 2e^{-2x}$$

$$y = -e^{-2x}\cos x + 2e^{-2x}$$

$$y = e^{-2x}(2 - \cos x)$$

7) $$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$$

$3m^2 - 2m - 1 = 0$ (LHS = 0)

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 3,\ b = -2,\ c = -1$

$$= \frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times -1}}{2(3)}$$

$$= \frac{2 \pm \sqrt{4 + 12}}{6}$$

$$= \frac{2 \pm \sqrt{16}}{6} = \frac{2 \pm 4}{6}, \quad \frac{2+4}{6} = 1$$

$$, \quad \frac{2-4}{6} = -1/3$$

$$y = Ae^{-x/3} + Be^{3}$$

R.H.S

$f(x) = 2x - 3$

$y = Cx + D$

$y' = C$

$y'' = 0$

Substituting

$3(0) - 2c - cx - D = 2x - 3$

$-2c - cx - D = 2x - 3$

$-cx - 2c - D = 2x - 3$

$-cx = 2x$	$-2c - D = -3$	$y = -$
$c = -2$	$-2(-2) - D = -3$	
	$4 - D = -3$	
	$D = -7$	

$y = -2x - 7$

$GS = CF + PI$

$y = Ae^{-x/3} + Be^{x} - 2x - 7$

8) $d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{4x}$

$y'' - 6y' + 8y = 0$

$k^2 - 6k + 8 = 0$

$k_1 = 2,\ k_2 = 4$

$CF = Ae^{2x} + Be^{4x}$

PI, $f(x) = 8e^{4x}$

$y = Cxe^{4x}$

$\cancel{y' = 4ce^{4x},\ y'' = 16}$

$y' = c[x(4e^{4x}) + e^{4x}(1)]$

$y' = c[4xe^{4x} + e^{4x}] = 4cxe^{4x} + ce^{4x}$

$y'' = c[16xe^{4x} + 4e^{4x} + 4e^{4x}]$

$y'' = c[16xe^{4x} + 8e^{4x}]$

$= 16cxe^{4x} + 8ce^{4x}$

Substituting

$(16cxe^{4x} + 8ce^{4x}) - 6(4cxe^{4x} + ce^{4x}) + 8cxe^{4x} = 8e^{4x}$

$16cxe^{4x} + 8ce^{4x} - 24cxe^{4x} - 6ce^{4x} + 8cxe^{4x} = 8e^{4x}$

$8ce^{4x} - 6ce^{4x} = 8e^{4x}$

$2c\cancel{e^{4x}} = 8\cancel{e^{4x}}$

$2c = 8 \quad \therefore c = 4$

$PI = 4xe^{4x}$

$\therefore y = Ae^{2x} + Be^{4x} + 4xe^{4x}$