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15/ENG06/013
MECHANICAL ENGINEERING

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Auxiliary equation becomes

$m^2 - m - 2 = 0$

$m^2 - m + 2m - 2 = 0$

$m(m-1) + 2(m-1) = 0$

$(m-1)(m+2) = 0$

$\therefore m = 1$ or -2

The complementary equation becomes

$y = Ae^{x} + Be^{-2x}$

To find the Particular Integral

Assumed PI, $y = C$ — ①

$dy/dx = 0$ — ②

$d^2y/dx^2 = 0$ — ③

Substitute equ ①, ②, ③ into the original equation

$0 - 0 - 2(c) = 8$

$-2c = 8$

$c = \frac{8}{-2} = -4$

$\therefore$ The general solution of the equation = Complementary Function + Particular Integral

$y = Ae^{x} + Be^{-2x} - 4$

2. ~~$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$~~

~~Auxiliary equation becomes~~

~~$m^2 - 4m = 0$~~

~~$m(m-4) = 0$~~

~~$m = 0$ or $m = 4$~~

~~The complementary equation becomes~~

~~$y =$~~

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

The auxiliary equation becomes

$m^2 - 4 = 0$

$m^2 = 4$

$m = \sqrt{4}$

$m = \pm 2$ $(2$ or $-2)$

The complementary Function becomes

$y = Ae^{2x} + Be^{-2x}$

To find the Particular Integral

Assumed PI, $y = Ce^{3x}$

$10e^{3x}(e^{kx}) = Ce^{kx}$

$\therefore y = Ce^{3x}$ — ①

$dy/dx = 3Ce^{3x}$ — ②

$d^2y/dx^2 = 9Ce^{3x}$ — ③

Substitute equ ①, ②, ③ into the original equation

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$5C = 10$

$C = \frac{10}{5} = 2$

$\therefore$ The general solution = Complementary Function + Particular Integral

$y = Ae^{2x} + Be^{-2x} + Ce^{3x}$

$= Ae^{2x} + Be^{-2x} + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

The auxiliary equation becomes

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1)=0$

$m=-1$ twice

The complementary equation becomes

$y=e^{-x}(A+Bx)$

To find the particular Integral

$(e^{-2x})e^{kx}=Ce^{kx}$

$\therefore y=Ce^{-2x}$ — ①

$dy/dx=-2Ce^{-2x}$ — ②

$d^2y/dx^2=4Ce^{-2x}$ — ③

Substitute equ ①, ②, ③ into the original equation

$4Ce^{-2x}+2(-2Ce^{-2x})+Ce^{-2x}=e^{-2x}$

$4Ce^{-2x}-4Ce^{-2x}+Ce^{-2x}=e^{-2x}$

$Ce^{-2x}=e^{-2x}$

$\therefore C=1$

$\therefore$ Assumed $PI=Ce^{-2x}$

$=e^{-2x}$

The general solution becomes

$y=e^{-x}(A+Bx)+e^{-2x}$

4) $\frac{d^2y}{dx^2}+25y=5x^2+x$

Auxiliary equation becomes

$m^2+25=0$

$m^2=-25$

$m=\sqrt{-25}$

$m=\pm\sqrt{-1}\times\sqrt{25}$

$m=\pm5i$

$m=\pm\beta j$

$\therefore \beta=5\ (\alpha=0)$

The complementary function becomes

$y=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

$y=A\cos5x+B\sin5x$

To find the assumed PI

$(5x^2+x)=Cx^2+Dx+E$

$\therefore y=Cx^2+Dx+E$ — ①

$dy/dx=2Cx+D$ — ②

$d^2y/dx^2=2C$ — ③

Substitute equ ①, ②, 3 into the original equation

$2C+25(Cx^2+Dx+E)=5x^2+x$

$2C+25Cx^2+25Dx+25E=5x^2+x$

Comparing coefficients

$25C=5$ — ①

$C=5/25=1/5$

$25D=1$ — ②

$D=\frac{1}{25}$

$2C+25E=0$ — ③

$2(\frac{1}{5})=-25E$

$E=\frac{2}{5}\div-25=\frac{2}{5}\times\frac{1}{-25}=\frac{-2}{125}$

$\therefore y=Cx^2+Dx+E$

$y=\frac{x^2}{5}+\frac{x}{25}-\frac{2}{125}$

The general solution becomes

$y=A\cos5x+B\sin5x+\frac{x^2}{5}+\frac{x}{25}-\frac{2}{125}$

5) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=2e^{-2x}4\sin x$

The auxiliary equation becomes

$m^2-2m+1=0$

$m^2-m-m+1=0$

$m(m-1)-1(m-1)=0$

$(m-1)(m-1)=0$

$m=1$ twice

The complementary function becomes

$y = e^{x}(A+Bx)$

To find assumed PI

$y(4\sin x) = C\cos x + D\sin x$

$y = C\cos x + D\sin x$ — ①

$dy/dx = -C\sin x + D\cos x$ — ②

$d^2y/dx^2 = -C\cos x - D\sin x$ — ③

Substitute equ ①, ② and ③ into the original equation

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x)$
$+ C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x$
$+ D\sin x = 4\sin x$

$2C\sin x - 2D\cos x = 4\sin x$

Comparing coefficients

$2C = 4$ — ①

$C = 4/2 = 2$

$2D\cos x = 0$ — ②

$D = 0$

$\therefore y = C\cos x + D\sin x$

$y = 2\cos x$

The solution to the general equation becomes $y = e^{x}(A+Bx) + 2\cos x$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

$x = 0$, $y = 1$ and $dy/dx = -2$

The auxiliary equation becomes

$m^2 + 4m + 5 = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$$

$$m = \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$$

$$m = \frac{-4 \pm \sqrt{-1} \times \sqrt{4}}{2} = \frac{-4 \pm \sqrt{-1} \times 2}{2}$$

$m = -2 \pm j$

Comparing with $m = \alpha \pm \beta j$

$\alpha = -2$, $\beta = 1$

The complementary function equation becomes $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$y = e^{-2x}(A\cos x + B\sin x)$

To find assumed PI,

$2e^{-2x}(Ce^{kx}) = Ce^{kx}$

$\therefore y = Ce^{-2x}$ — ①

$dy/dx = -2Ce^{-2x}$ — ②

$d^2y/dx^2 = 4Ce^{-2x}$ — ③

Substitute equ ①, ②, ③ into the original equation

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x})$
$= 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x} = 2e^{-2x}$

$C = 2$

$\therefore$ Assumed $PI = 2e^{-2x}$

The general solution becomes

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

when $x = 0$, $y = 1$

$1 = e^{-2(0)}(A\cos 0 + B\sin 0) + 2e^{-2(0)}$

$1 = A + 2$

$1 - 2 = A$

$\therefore A = -1$

when $x = 0$, $dy/dx = -2$

$dy/dx = -2e^{-2x}(-A\sin x + B\cos x) - 4e^{-2x}$

$\therefore -2 = -2e^{-2(0)}(-A\sin 0 + B\cos 0)$
$- 4e^{-2(0)}$

$-2 = B - 4$

$B = -2+4$

$B = 2$

$\therefore$ The general solution to the equation becomes

$y = e^{-2x}(-\cos x + 2\sin x) + 2e^{-2x}$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x-3$

The auxiliary equation becomes

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(3m+1)(m-1) = 0$

$3m+1 = 0$ or $m = 1$

$m = -1/3$ or 1

$\therefore$ The complementary function becomes $y = Ae^{x} + Be^{-1/3 x}$

To find assumed PI

$f_x\ (-2x-3) = Cx + D$

$y = Cx + D$ — ①

$dy/dx = C$ — ②

$d^2y/dx^2 = 0$ — ③

Substitute the ~~original~~ equ ①, ②, ③ into the original equation

$3(0) - 2(C) - (Cx+D) = 2x-3$

$-2C - Cx - D = 2x - 3$

Comparing coefficients

$-C = 2$ — ①

$\therefore C = -2$

$-2C - D = -3$ — ②

$-2(-2) - D = -3$

$4 - D = -3$

$4 + 3 = D$

$D = 7$

Assumed PI $= -2x + 7$

The general solution to the equation becomes $y = Ae^{x} + Be^{-1/3 x} - 2x + 7$

e) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

The auxiliary equation becomes

$m^2 - 6m + 8 = 0$

$m^2 - 2m - 4m + 8 = 0$

$m(m-2) - 4(m-2) = 0$

$(m-4)(m-2) = 0$

$m = 4$ or $m = 2$

The complementary function solution becomes $y = Ae^{4x} + Be^{2x}$

To find Assumed PI

$8e^{4x}\ (Ce^{kx}) = Ce^{kx}$

$\therefore y = Ce^{4x}$ — ①

$dy/dx = 4Ce^{4x}$ — ②

$d^2y/dx^2 = 16Ce^{4x}$ — ③

Substitute equ ①, ② & ③ into the original equation

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

$0 = 8e^{4x}$

multiply equ ① (Ce^{kx}) by x

$\therefore y = Cxe^{4x}$ — ④

$dy/dx = e^{4x}(C) + Cx(4e^{4x})$ — ⑤

$d^2y/dx^2 = 4Ce^{4x} + e^{4x}(4C) + 4Cx(4e^{4x})$

$= 4Ce^{4x} + 4Ce^{4x} + 16Cxe^{4x}$ — ⑥

Substitute equ ④, ⑤, ⑥ into the original equation

$8Ce^{4x} + 16Cxe^{4x} - 6(Ce^{4x} + 4Cxe^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

divide through by e^{4x}

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$8c + 16cx - 6c - 24cx + 8cx = 8$

$2c \cancel{+ 12cx} = 8$

$\cancel{c = 6cx = 4}$ $\therefore c = 8/2 = 4$

$\therefore$ Assumed $PI = Cxe^{4x} = 4xe^{4x}$

The general solution to the equation becomes $y = Ae^{4x} + Be^{2x} + 4xe^{4x}$