16 ENG403/059 FRENCH ERERE

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2 = 0$

$(m^2 - 2m)(m-2) = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2) = 0$

$m = -1$ and 2

$y = Ae^{2x} + Be^{-x}$

$y = c$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$0 - 0 - 2c = 8$

$-2c = 8$

$c = -4$

$PI = y = -4$

$y = Ae^{2x} + Be^{-x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = \pm 4$

$m = \pm\sqrt{4}$

$m = \pm 2$

$C.F = A\cosh 2x + B\sinh 2x$

$P.I = y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$9Ce^{3x} + Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$C = 2$

$y = 2e^{3x}$

$G.S = Ae^{2x} + Be^{-2x} + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4 = e^{-2x}$

$m^2 + 2m + 1 = 0$

$(m^2 + m) + (m + 1) = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1$ (twice)

$y = e^{-x}(A + Bx)$

$P.I = y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$= 4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$y = e^{-2x}$

$G.S = e^{-x}(A + Bx) + e^{-2x}$

$m = \pm 5$

$C.F = A\cos 5x + B\sin 5x$

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{dy2}{dx} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$2C + 25Cx^2 + 25Dx = 5x^2 + x$

$25C = 5$

$C = 5/25 = 1/5$

$25D = 1$

$D = \frac{1}{25}$

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$2/5 = -25E$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$m = 1$ twice

$y = e^x(A + Bx)$

$y = C\cos x + A\sin x$

$\frac{dy}{dx} = -C\sin x + A\cos x$

$\frac{d^2y}{dx^2} = C\cos x - A\sin x$

$= -C\cos x - A\sin x - 2(-C\sin x + A\cos x) + C\cos x + A\sin x = 4\sin x$

$-C\cos x - A\sin x + 2C\sin x - 2A\cos x + C\cos x + A\sin x = 4\sin x$

$= 2C\sin x - 2A\cos x = 4\sin x$

compare both equation

$2C = 4$

$C = 2$

$-2A = 0$

$A = 0$

$y = C\cos x + A\sin x$

$$\frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm j\sqrt{4}}{2}$$

$$m = -\frac{4}{2} + i\frac{\sqrt{4}}{2} = -2 \pm j$$

$\alpha = -2$, $\beta = 1$

C.F $= y = e^{-2x}(A\cos x + B\sin x)$

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$= 4Ce^{-2x} - 2Ce^{-2x}(4) + 5Ce^{-2x} = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x} = 2e^{-2x}$

$C = 2$

$y = 2e^{-2x}$

G.S $= e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

Given that $x = 0, y = 1$ and $\frac{dy}{dx} = -2$

Q7

$y = Ae^{-2x}\cos x + Be^{-2x}\sin x + 2e^{-2x}$

$1 = Ae^{-2(0)}\cos 0 + Be^{-2(0)}\sin 0 + 2e^{-2(0)}$

$1 = A + 2$

$A = 1 - 2 = -1$

$\frac{dy}{dx} = -2Ae^{-2x}\cos x - \sin x Ae^{-2x} - 2Be^{-2x}\sin x + \cos x Be^{-2x} - 4e^{-2x}$

$-2 = -2Ae^{-2(0)} + Be^{-2(0)} - 4e^{-2(0)}$

$-2 = -2A + B - 4$

$-2 = -2(-1) + B - 4$

$-2 = +2 + B - 4 \quad = -2 + 2 = B$

$B = 0$

$y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-x}$

$y = e^{-2x}(0\sin x + 2e^{-2x} = e^{-2x}(-\cos x + 2)$

1) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 4y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$(3m^2 - 3m)(m - 1) = 0$

$3m(m - 1) + 1(m - 1) = 0$

$(3m + 1)(m - 1) = 0$

$m = -1/3$

$y = Ae^{-1/3x} + Be^{x}$

P.I $= y = kx^0 B(x + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 4y = 2x - 3$

$3(0) - 2(x \cdot -C(x + D) = 2x - 3$

$-2Cx - C(x - D = 2x - C$

compare the terms

$-Cx = 2x$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$-D = -3 - 4$

$-D = -7$

$D = 7$

$y = -2x + 7$

G.S $= Ae^{-x/3} + Be^{x} - 2x + 7$

$(m^2-4m)(-2m+8)=0$

$m(m-4)-2(m-4)=0$

$(m-2)(m-4)=0$

$m=2$ and 4

$y=Ae^{2x}+Be^{4x}$

$y=ce^{4x}$

$\frac{dy}{dx}=4ce^{4x}$

$\frac{d^2y}{dx^2}=16ce^{4x}$

$=16ce^{4x}-6(4ce^{4x})+8(ce^{4x})=8e^{4x}$

$16ce^{4x}-24ce^{4x}+8ce^{4x}=8e^{4x}$

c is undetined

therefore $y=cxe^{4x}$

product rule should be used to differentiate

$u=cx \quad v=e^{4x}$

$\frac{du}{dx}=c \quad \frac{dv}{dx}=4e^{4x}$

$\frac{dy}{dx}=cx(4e^{4x})+e^{4x}(c)$

$\frac{dy}{dx}=4cxe^{4x}+ce^{4x}$

$\frac{d^2y}{dx^2}=16cxe^{4x}+4ce^{4x}+4ce^{4x}$

$=16cxe^{4x}+8ce^{4x}$

$=16cxe^{4x}+8ce^{4x}-6(4cxe^{4x}+ce^{4x})+8cxe^{4x}=8e^{4x})$

$=16cxe^{4x}+8ce^{4x}-24cxe^{4x}-6ce^{4x}+8cxe^{4x}=8e^{4x}$

$2ce^{4x}=8e^{4x}$

$c=4$

$y=4xe^{4x}$

Eq. 3 $=Ae^{2x}+Be^{4x}+4xe^{4x}$