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15/ENG04/011

Elect - Elect Engr.

ENG 381 Assignment 1

(1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Soln

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m^2 + m - 2m - 2 = 0$

$m(m+1) - 2(m+1) = 0$

$m - 2 = 0$ or $m + 1 = 0$

$m_1 = 2 \qquad m_2 = -1$

$y = Ae^{2x} + Be^{-x} \Rightarrow$ C F

To obtain P.I

$y = C$

$\frac{dy}{dx} = 0 \qquad \frac{d^2y}{dx^2} = 0$

$0 - 0 - 2C = 8$

$-2C = 8 \quad \therefore C = -4 \quad \therefore y = -4$

G.S = C.F + P.I

$y = Ae^{2x} + Be^{-x} - 4$

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2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2$ $\quad m_1 = 2 \quad m_2 = -2$

$y = C\cosh 2x + D\sinh 2x$ => C.F

To obtain P.I

$y = Ae^{3x}$

$\frac{dy}{dx} = 3Ae^{3x}$ $\quad \frac{d^2y}{dx^2} = 9Ae^{3x}$

$9Ae^{3x} - 4(Ae^{3x}) = 10e^{3x}$

$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$Ae^{3x}(9-4) = 10e^{3x}$

$A = \frac{10e^{3x}}{5e^{3x}} = 2$

$y = 2e^{3x}$ - - P.I

G.S = $y = C\cosh 2x + D\sinh 2x + 2e^{3x}$

3. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

Soln

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$m_1 = m_2 = -1$

$y = e^{-x}(A + Bx) \longrightarrow$ C.F

to obtain P.I

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$ $\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x}[4 - 4 + 1] = e^{-2x}$

$C = \frac{e^{-2x}}{e^{-2x}} = 1$

$\therefore y = e^{-2x}$ --- P.I

G.S: $y = e^{-x}(A + Bx) + e^{-2x}$

4. $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25 \quad \therefore m = \pm\sqrt{-25}$

$m = \pm\sqrt{-1}\sqrt{25} \quad \therefore m = \pm j5$

$y = C\cos 5x + D\sin 5x$ — CF

to obtain P.I

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D \qquad \frac{d^2y}{dx} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 + 25Dx + 25E + 2C = 5x^2 + x$

Comparing Coefficients

$25C = 5$ — (i)

$C = \frac{1}{5}$

$25D = 1$ — (ii)

$D = \frac{1}{25}$

$25E + 2C = 0$ — (iii)

$2(\frac{1}{5}) = -25E$

$\cancel{E = -\frac{1}{25}}$ $\quad E = \frac{2}{5} \div -25 = -\frac{2}{125}$

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$\therefore y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$ = P.I

G.S: $y = C\cos 5x + D\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

5. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1) = 0$

$m_1 = m_2 = 1$

$y = e^x(A + Bx)$ — -- CF

to obtain P.I

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$(-C\cos x - D\sin x) - 2(-C\sin x + D\cos x) +$
$C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x$
$+ D\sin x = 4\sin x.$

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$-D\sin x + 2C\sin x + D\sin x - C\cos x - 2D\cos x +$
$C\cos x = 4\sin x$

Comparing Co-efficients

$-\cancel{D} + 2C + \cancel{D} = 4 \quad --① $

$2C = 4 \quad \therefore C = 2$

$-\cancel{C} - 2D + \cancel{C} = 0$

$-2D = 0 \quad \therefore D = 0$

$y = 2\cos x + 0\sin x = 2\cos x \rightsquigarrow P.I$

G.S $= y = e^x(A + Bx) + 2\cos x$

6. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1}$

$m = \frac{-4 \pm \sqrt{16 - 4}}{2}$

$m = \frac{-4 \pm \sqrt{-1}\sqrt{4}}{2} = \frac{-4 \pm j2}{2}$

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$m = -2 \pm j \quad \therefore \quad m_1 = -2 + j \quad m_2 = -2 - j$

$y = e^{-2x}(C\cos x + D\sin x) \Rightarrow C.F$

$\vdash$ obtain P.I

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x} \qquad \frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x}(4 - 8 + 5) = 2e^{-2x}$

$C = \frac{2e^{-2x}}{e^{-2x}} = 2$

$y = 2e^{-2x}$ — — P.I

G.S $= y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$

at $x = 0 \quad y = 1 \quad \frac{dy}{dx} = -2$

$1 = e^0(C\cos 0 + \cancel{D\sin 0}) + 2e^0$

$1 = C + 2 \quad \therefore \quad C = -1$

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$$\frac{dy}{dx} = e^{-2x}(-C\sin x + D\cos x) + [C\cos x + D\sin x] \times -2e^{2x} - 4e^{2x}$$

at $x = 0$ $y = 1$ $\frac{dy}{dx} = -2$

$$-2 = e^0[-C\cancel{\sin 0}^{0} + D\cos 0] + [C\cos 0 + \cancel{D\sin 0}^{0}] \cdot -2e^0 - 4e^0$$

$$-2 = D - 2C - 4$$

since $C = -1$

$$-2 = D - 2(-1) - 4$$

$$-2 = D + 2 - 4$$

$$-2 + 2 = D$$

$$D = 0$$

P.S = $y = e^{-2x}[-\cos x] + 2e^{-2x}$

$$\therefore y = -e^{-2x}\cos x + 2e^{-2x}$$

7. $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$

$3m^2 - 2m - 1 = 0$

$\cancel{3m^2 + \frac{1}{3}m - m - 1 = 0}$

$\cancel{B}C$

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(3m+1)(m-1) = 0$

$m_1 = -\frac{1}{3} \quad m_2 = 1$

$y = Ae^{-\frac{1}{3}x} + Be^{x}$ — C.f

to obtain P.I

$y = Cx + D$

$$\frac{dy}{dx} = C$$

$$\frac{d^2y}{dx^2} = 0$$

$$0 - 2C - (Cx + D) = 2x - 3$$

$$-2C - Cx - D = 2x - 3$$

Comparing coefficients

$$-C = 2$$

$$C = -2$$

$$-2C - D = -3$$

$$-2(-2) - D = -3$$

$$4 - D = -3$$

$$D = 7$$

$$y = -2x + 7$$

$$G.S = y = Ae^{-\frac{1}{3}x} + Be^{x} - 2x + 7$$

8. $$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$$

$$m^2 - 6m + 8 = 0$$

$$m^2 - 2m - 4m + 8 = 0$$

$m(m-2) - 4(m-2) = 0$

$(m-4)(m-2) = 0$

$m_1 = 4 \quad m_2 = 2$

$y = Ae^{4x} + Be^{2x}$ — — C.F

1- obtain P.I

$y = Cxe^{4x}$

$\frac{dy}{dx} = C[x \cdot 4e^{4x} + e^{4x}]$

$= 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 4C[x \cdot 4e^{4x} + e^{4x}] + 4Ce^{4x}$

$= 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$16Cxe^{4x} + 8Ce^{4x} - 6[4Cxe^{4x} + Ce^{4x}] +$
$8[Cxe^{4x}] = 8e^{4x}$

$16Cxe^{4x} + 8Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x}$
$= 8e^{4x}$

$Cxe^{4x}\underbrace{(16 - 24 + 8)}_{0} + Ce^{4x}(8-6) = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$C = \frac{8e^{4x}}{2e^{4x}} = 4$

$y = 4xe^{4x}$

G.S: $y = Ae^{4x} + Be^{2x} + 4xe^{4x}$