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Mechanical Engineering

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8.$

Auxillary equation becomes

$m^2 - m - 2 = 0.$

$m^2 - m + 2m - 2 = 0.$

$m(m-1) + 2(m-1) = 0.$

$(m-1)(m+2) = 0.$

The complementary equation becomes.

$y = Ae^{2} + Be^{-2x}$

To find the particular Integral

Assume PI, $y = c$ —— (1).

$\frac{dy}{dx} = 0$ — (2), $\frac{d^2y}{dx^2} = 0$ — (3).

substitute equ. 1, 2, 3 into original equation

$0 - 0 - 2c = 8$

$-2c = 8$

$c = 8/-2 = -4.$

The general solution to the equation = Complementary function (CF) + Particular Integral (PI)

$y = Ae^{x} + Be^{-2x}$ —4

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Auxillary equation becomes.

$m^2 - 4 = 0.$

$m^2 = 4$

$m = \pm 2$ (+2 or −2).

The complementary function becomes.

$y = Ae^{2x} + Be^{-2x}.$

To find the particular Integral

Assumed PI, $y = Ce^{2x}$

$10e^{3x}(e^{kx}) = ce^{kx}$

$\therefore y = Ce^{3x}$ —— (1).

$\frac{dy}{dx} = 3Ce^{3x}$ — (2), $\frac{d^2y}{dx^2} = 9Ce^{3x}$ — (3)

substitute eqn 1, 2, 3 into the original equation.

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{x} - 4Ce^{3x} = 10e^{3x}$

$5C\cancel{e^{3x}} = 10\cancel{e^{3x}}$

$5C = 10$

$C = 2.$

The general solution = Complementary function (CF) + Particular Integral (PI)

$y = Ae^{2x} + Be^{-2x} + Ce^{3x}$

$= Ae^{2x} + Be^{-2x} + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}.$

Auxillary equation becomes.

$m^2 + 2m + 1 = 0.$

$m^2 + m + m + 1 = 0$

$m[m+1] + 1[m+1].$

$[m+1](m+1) = 0.$

$m = -1$ twice.

The complementary equation becomes

$y = e^{-x}(A + Bx).$

To find the particular Integral.

$(e^{-2x})e^{kx} = Ce^{kx}$

$\therefore y = Ce^{-2x}$ —(1).

$dy/dx = -2Ce^{-2x}$ —(2).

$d^2y/dx^2 = 4Ce^{-2x}$ —(3)

substitute eqn 1,2,3 into the original eqn:

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$C\cancel{e^{-2x}} = \cancel{e^{-2x}}$

$C = 1.$

Assume PI $= Ce^{-2x}$

$= e^{-2x}$

the general solution becomes:

$y = e^{-x}(A+Bx) + e^{-2x}$

4). $\frac{d^2y}{dx^2} + 25y = 5x^2 + x.$

The auxillary equation becomes $m^2+25m=0$.

$m^2 = -25m$

$m^2 = \sqrt{-25}$.

$m^2 = \pm\sqrt{-1} \times \sqrt{25}$.

$m = \pm 5i$

$m = \pm \beta i$

$\therefore \beta = 5 \ (\alpha = 0).$

The Complementary function becomes

$y = e(A\cos\beta x + B\sin\beta x).$

$y = A\cos 5x + B\sin 5x.$

To find the assumed PI.

$(5x^2+x) = Cx^2 + Dx + E.$

$\therefore \ y = Cx^2 + Dx + E$ —(1).

$dy/dx = 2Cx + D$ —(2).

$d^2y/dx^2 = 2C$ —(3).

Substitute equ 1,2&3 into the Original eqn.

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x.$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x.$

Comparing coefficients.

$25C = 5$ —(1).

$C = 5/25 = 1/5$

$25D = 1$ —(2).

$D = 1/25.$

$2C + 25E = 0$ —(3).

$2(1/5) = -25E.$

$E = 2/5 \div -25 = 2/5 \times \frac{1}{-25} = -2/125.$

$\therefore \ y = Cx^2 + Dx + E.$

$y = x^2/5 + x/25 - 2/125.$

The general solution becomes.

$y = A\cos 5x + B\sin 5x + \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}.$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x.$

The auxiliary equation becomes.

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0.$

$m(m-1) - 1(m-1) = 0.$

$(m-1)(m-1) = 0.$

$m = 1$ twice

the complementary function becomes.

$y = e^x(A+Bx).$

To find assumed PI.

$y(4\sin x) = C\cos x + D\sin x.$

$y = C\cos x + A\sin x$ —(1).

$dy/dx = -C \sin x + D \cos x$ — (2)

$d^2y/dx^2 = -C\cos x - D \sin x$ — (3)

Substitute eqn 1, 2, 3 into the original eqn

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$2C\sin x - 2D\cos x = 4\sin x$

Comparing Co-efficients

$x = 4$ — (1)

$C = 4/2 = 2$

$2D\cos x = 0$ — (2)

$D = 0$

$\therefore y = C\cos x + D\sin x$

$y = 2\cos x$

The solution to the general equation becomes

$y = e^x(A + Bx) + 2\cos x$

The solution to the general equation becomes

$y = e^x(A + Bx) + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that

$x = 0, y = 1$ and $dy/dx = -2$

The auxillary equation becomes

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm \sqrt{-1} \times \sqrt{4}}{2} = \frac{-4 \pm \sqrt{-1} \times 2}{2}$

$m = -2 \pm j$

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comparing with $m = \alpha + \beta i$

$\alpha = -2, \beta = 1.$

The complementary function equation becomes

$y = e^{2x}(A\cos\beta x + B\sin\beta x).$

$y = e^{2x}(A\cos x + B\sin x).$

To find assumed PI.

$2e^{-2x}(ce^{kx}) = ce^{kx}.$

$\therefore y = ce^{-2x}$ —(1).

$dy/dx = -2ce^{-2x}$ —(2)

$d^2y/dx^2 = 4ce^{-2x}$ —(3).

substitute equ 1,2,3 into the original eqn

$4ce^{-2x} + 4(-2ce^{-2x}) + 5(ce^{-2x}) = 2e^{-2x}$

$4ce^{-2x} - 8ce^{-2x} + 5ce^{-2x} = 2e^{-2x}$

$ce^{-2x} = 2e^{-2x}$

$c = 2.$

Assumed PI $= 2e^{-2x}$

The general solution becomes

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

when $x = 0$ & $y = 1.$

$1 = e^{-2(0)}(A\cos 0 + B\sin 0) + 2e^{-2(0)}$

$1 = A + 2.$

$1 - 2 = A$

$A = -1.$

when $x = 0$, $dy/dx = -2.$

$dy/dx = -2e^{-2x}(A\sin x + B\cos x) - 4e^{-2x}$

$\therefore -2 = -2e^{-3(0)}(A\sin 0 + B\cos 0) - 4e^{-2(0)}$

$-2 = B - 4.$

$B = -2 + 4 = 2.$

The general solution becomes-

$y = e^{-2x}(-\cos x + 2\sin x) + 2e^{-2x}$

7) $3\,d^2y/dx^2 - 2\,dy/dx - y = 2x - 3.$

The auxillary equation becomes

$3m^2 - 2m - 1 = 0.$

$3m^2 - 3m + m - 1 = 0.$

$3m(m-1) + 1(m-1) = 0.$

$(3m+1)(m-1) = 0.$

$(3m+1)(m-1) = 0.$

$3m + 1 = 0$ or $m = 1.$

$m = -1/3$ or $1.$

$\therefore$ The complementary function become

$y = Ae^{x} + Be^{-1/3x}$

To find assumed PI.

$f(x)(-2x-3) = Cx + D.$

$y = Cx + D$ —(1).

$dy/dx = C$ —(2).

$d^2y/dx^2 = 0$ —(3).

Sub equation 1,2,3 into the original eqn

$3(0) - 2(c) - (cx + D) = 2x - 3.$

$-2c - cx - D = 2x - 3.$

Comparing coefficients.

$-c = 2$ —(1).

$\therefore c = -2.$

$-2c - D = -3$ —(2).

$-2(-2) - D = -3$

$4 - D = -3.$

$4 + 3 = D.$

$D = 7.$

Assumed PI $= -2x + 7.$

The general solution to the equation becomes $y = Ae^{x} + Be^{-1/3x} - 2x + 7.$

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$.

The auxiliary eqn becomes:

$m^2 - 6m + 8 = 0$.

$m^2 - 2m - 4m + 8 = 0$.

$m(m-2) - 4(m-2) = 0$.

$(m-4)(m-2) = 0$.

$m = 4$ or $m = 2$.

The Complementary function solution becomes $y = Ae^{4x} + Be^{2x}$.

To find Assumed PI.

$8e^{4x}$ $(e^{kx}) = Ce^{kx}$

$\therefore y = Ce^{4x}$ —(1).

$dy/dx = 4Ce^{4x}$ —(2).

$d^2y/dx^2 = 16Ce^{4x}$ —(3).

Substitute equ (1),(2)(3) into the original equation.

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

$0 = 8e^{4x}$

Multiply eqn (1) (Ce^{kx}) by x.

$\therefore y = Cxe^{4x}$ —(4).

$dy/dx^2 = e^{4x}(c) + cx(4e^{4x})$ —(5).

$d^2y/dx^2 = 4Ce^{4x} + e^{4x}(4c) + 4cx(4e^{4x})$.

$= 4Ce^{4x} + 4Ce^{4x} + 6Ce^{4x}$ —(6)

Substitute eqn 4, 5, 6 into the Original equation.

$8Ce^{4x} + 16Cxe^{4x} - 6(Ce^{4x} + 4Cxe^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

divide through by e^{4x}.

$8C + 16Cx - 6C - 24Cx + 8Cx = 8$.

$x = 8$.

$C = 8/2 = 4$.

Assumed PI $= Cxe^{4x} = 4xe^{4x}$.

The general solution to the equation becomes

$y = Ae^{4x} + Be^{2x} + 4xe^{4}$