(i) [illegible] $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 8$

Auxiliary equation becomes

$m^2 + m - 2 = 0$

$m^2 + 2m - m - 2 = 0$

$m(m+2) - 1(m+2) = 0$

$(m-1)(m+2) = 0$

$m = 1$ or $m = -2$

The complementary function becomes

$y = Ae^{x} + Be^{-2x}$

To find the particular Integral

Assumed PI $y = C$ —(1)

$dy/dx = 0$ —(2)

$d^2y/dx^2 = 0$ —(3)

Substitute eqn (1)(2)(3) into the original eqn

$0 + 0 - 2(C) = 8$

$-2C = 8$

$C = \frac{-8}{2} = -4$

$\therefore$ The general equation of the equation

$= CF + PI$

$y = Ae^{x} + Be^{-2x} - 4$

(ii) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

The auxiliary equation becomes

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2$ (2 or −2)

The complementary function becomes

$y = Ae^{2x} + Be^{-2x}$

To find the particular Integral

Assumed PI $y = Ce^{3x}$ —(1)

$dy/dx = 3Ce^{3x}$ —(2)

$d^2y/dx^2 = 9Ce^{3x}$ —(3)

Substitute eqn (1)(2)(3) into the original equation

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$5C = 10$

$C = \frac{10}{5} = 2$

$\therefore$ The general solution =

CI + PF

$y = Ae^{2x} + Be^{-3x} + Ce^{3x}$

$= Ae^{2x} + Be^{-2x} + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

The auxiliary equation becomes

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1$ twice

The complementary equation becomes

$y = e^{-x}(A + Bx)$

To find the particular integral

$y = Ce^{-2x}$ —(1)

$dy/dx = -2Ce^{-2x}$ —(2)

$d^2y/dx^2 = 4Ce^{-2x}$ —(3)

Sub (1) (2) (3) in the original equation

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1$

Assume PI $= Ce^{-2x}$

$= e^{-2x}$

The general solution becomes

$y = e^{-x}(A + Bx) + e^{-2x}$

(4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Auxiliary equation becomes

$m^2 + 25 = 0$

$m^2 = -25$

$m = \sqrt{-25}$

$\sqrt{-25} = \sqrt{-1} \times \sqrt{25}$

$= j \times 5$

$= 5j$

$m = \pm 5j$

$m = \pm \beta j$

$\therefore \beta = 5 \quad (\alpha = 0)$

The complementary function becomes

$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$y = A\cos 5x + B\sin 5x$

To find the PI?

$(5x^2 + x) = Cx^2 + Dx + E$

$\therefore y = Cx^2 + Dx + E$ —(1)

$\frac{dy}{dx} = 2Cx + D$ —— (2)

$\frac{d^2y}{dx^2} = 2C$ —— (3)

Sub (1)(2)(3) into the original equ

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

Comparing coefficient

$25C = 5$

$C = \frac{5}{25} = \frac{1}{5}$

$25D = 1$

$D = \frac{1}{25}$

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$\frac{2}{5} + 25E = 0$

$E = \frac{2}{5} \div -25 = \frac{2}{5} \times \frac{1}{25} = \frac{-2}{125}$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

The auxiliary equation becomes

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) -1(m-1) = 0$

$(m-1)(m-1) = 0$

$m = 1$ twice

The complementary function becomes

$y = e^{x}(A + Bx)$

To find assumed PI

$y(\text{PI}) = C\cos x + D\sin x$ — (1)

$y = C\cos x + D\sin x$ — (1)

$dy/dx = -C\sin x + D\cos x$ — (2)

$d^2y/dx^2 = -C\cos x - D\sin x$ — (3)

Substitute (1)(2)(3) in the original equation

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x$
$+ D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x +$
$D\sin x = 4\sin x$

$2C\sin x - 2D\cos x = 4\sin x$

Comparing coefficient

$2C = 4$

$C = 4/2 = 2$

$2D\cos x = 0$

$D = 0$

$y = C\cos x + D\sin x$

$y = 2\cos x$

The solution to the general equation becomes $y = e^{x}(A + Bx) + 2\cos x$

(b) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$x = 0$, $y = 1$ and $d^2y/dx^2 = -2$

The auxiliary equation becomes

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm \sqrt{-1} \times \sqrt{4}}{2} = \frac{-4 \pm \sqrt{-1} \times 2}{2}$

$m = -2 \pm j$

Comparing with $m = \alpha \pm \beta j$

$\alpha = -2$, $\beta = 1$

The complementary function equation becomes $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$y = e^{-2x}(A\cos x + B\sin x)$

To find assumed PI

$y = Ce^{-2x}$ — (1)

$dy/dx = -2Ce^{-2x}$ — (2)

$d^2y/dx^2 = 4Ce^{-2x}$ — (3)

Substitute eqn (1)(2)(3) into the original equation

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x})$
$= 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x} = 2e^{-2x}$

$C = 2$

$\therefore$ Assumed PI $= 2e^{-2x}$

The general solution becomes

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

when $x = 0$, $y = 1$

$1 = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-2(0)}$

$1 = A + 2$

$A = 1 - 2$

$A = -1$

when $x = 0$ $dy/dx = 2$

$dy/dx = -2e^{-2x}(\ldots) (-A\sin x + B\cos x) - 4e^{-2x}$

$-2 = -2e^{-2(0)}(-A\sin(0) + B\cos(0)) - 4e^{-2(0)}$

$-2 = 2 - 4$

$B = -2 + 4$

$B = 2$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x-3$

The auxiliary equation becomes

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(3m+1)(m-1) = 0$

$3m+1 = 0$ or $m = 1$

$m = -1/3$ or 1

∴ The complementary function becomes

$y = Ae^{x} + Be^{-1/3x}$

To find PI

$y = Cx + D$ ——— (1)

$dy/dx = C$ ——— (2)

$d^2y/dx^2 = 0$ ——— (3)

Substitute the original equ (1)(2)(3) into the original equation

$3(0) - 2(C) - 1(Cx+D) = 2x-3$

$0 - 2C - Cx - D = 2x-3$

Comparing coefficient

$-C = 2$

∴ $C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$-D = -3 + 4$

$-D = -7$

$D = 7$

Assumed PI $= -2x + 7$

The auxiliary equation becomes

$y = Ae^{x} + Be^{-1/3x} - 2x + 7$

(8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

The auxiliary equation becomes

$m^2 - 6m + 8 = 0$

$m^2 - 2m - 4m + 8 = 0$

$m(m-2) - 4(m-2) = 0$

$(m-4)(m-2) = 0$

$m = 4$ or 2

The complementary function becomes

$y = Ae^{4x} + Be^{2x}$

To find assumed PI

$y = Ce^{4x}$ — (1)

$dy/dx = 4Ce^{4x}$ — (2)

$d^2y/dx^2 = 16Ce^{4x}$ — (3)

Substitute (1)(2)(3) into the original equation

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

$0 = 8e^{4x}$

Multiply equ (1) (Ce^{4x}) by x

∴ $y = Cxe^{4x}$ — (4)

$dy/dx = e^{4x}(C) + Cx(4e^{4x})$ — (5)

$d^2y/dx^2 = 4Ce^{4x} + e^{4x}(4C) + 4Cx(4e^{4x})$

$= 4Ce^{4x} + 4Ce^{4x} + 16Cxe^{4x}$ — (6)

Substitute equ (4)(5)(6) into the original equation

$8Ce^{4x} + 16Cxe^{4x} - 6(Ce^{4x} + 4Cxe^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

divide through by e^{4x}

$8C + 16Cx - 6C - 24Cx + 8Cx = 8$

$2C = 8$

∴ $C = 8/2 = 4$

∴ Assumed PI $= Cxe^{4x} = 4xe^{4x}$

The general ~~equation~~ solution to the equation becomes

$y = Ae^{4x} + Be^{2x} + 4xe^{4x}$