ONOH IKECHUKWU CHINEDU
16/ENG06/087
MECHANICAL ENGINEERING
ENG 381 (ASSIGNMENT).

Q1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

C.F

$m^2 - m - 2 = 0$

$(m-2)(m+1) = 0$

$m = 2$ or $m = -1$

$y = Ae^{m_1 x} + Be^{m_2 x}$

$y = Ae^{2x} + Be^{-x}$

P.I

$y = C$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$= 0 - 0 - 2C = 8$

$2C = -8$

$C = -8/2$

$C = -4$

∴ The General solution

$G.S = C.F + P.I$

$= Ae^{2x} + Be^{-x} - 4$

Q2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

C.F.

$m^2 - 4 = 0$

$m^2 = 4$

$m = \sqrt{4}$

$m = \pm 2$.

$y = A\cosh 2x + B\sinh 2x$.

P.I.

$y = Ce^{3x}$.

$\frac{dy}{dx} = 3Ce^{3x}$.

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$= 9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$.

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

Dividing through by e^{3x}

$9C - 4C = 10$

$5C = 10$

$C = 2$

∴ The General equation becomes

$CF + PI$

i.e $= A\cosh 2x + B\sinh 2x + 2e^{3x}$.

Q3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$.

C.F

$m^2 + 2m + 1 = 0$

$(m+1)(m+1) = 0$

$m = -1$ twice

$y = e^{-x}(A + Bx)$

P.I.

$y = Ce^{-2x}$.

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$.

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$\cancel{4Ce^{-2x}} - \cancel{4Ce^{-2x}} + Ce^{-2x} = e^{-2x}$.

$Ce^{-2x} = e^{-2x}$

$C = 1$

$\therefore$ the general solution will be

$= e^{-x}(A + Bx) + e^{-2x}$

Q4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

C.F

$m^2 + 25 = 0$

$m^2 = -25$

$m = \sqrt{-25}$

$= \sqrt{-1}\sqrt{25}$

$m = j5$.

$y = A\cos nx + B\sin nx$.

$= A\cos 5x + B\sin 5x$.

P.I

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$\Rightarrow 2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$= 2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$.

Comparing coefficients:

$25C = 5$

$C = \frac{1}{5}$

$25D = 1$

$D = \frac{1}{25}$

$2C + 25E = 0$

$2(\frac{1}{5}) + 25E = 0$

$25E = -\frac{2}{5}$

$E = -\frac{2}{125}$

Rit

$\therefore$ The general solution becomes

C.F + P.I

i.e $A\cos 5x + B\sin 5x + \frac{25x^2 + 5x - 2}{125}$

Q5)

(25) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x.$

C.F

$m^2 - 2m + 1 = 0$

$(m-1)(m-1) = 0$

$m = 1$ twice

$y = e^x(A + Bx).$

P.I

$y = C\cos x + D\sin x.$

$\frac{dy}{dx} = -C\sin x + D\cos x.$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x.$

$\Rightarrow -C\cos x - D\sin x - 2(-C\sin x + D\cos x)$
$+ C\cos x + D\sin x = 4\sin x$

$= -C\cos x - D\sin x + 2C\sin x - 2D\cos x$
$+ C\cos x + D\sin x = 4\sin x.$

$2C\sin x - 2D\cos x = 4\sin x$

Comparing Coefficients:

$2C = 4$

$C = 4/2 \Rightarrow C = 2$

$2D = 0$

$D = 0.$

P.I $y = 2\cos x + 0\sin x = 2\cos x$

General solution become

$e^{x}(A + Bx) + 2\cos x$

(26) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

given that at $x = 0$, $y = 1$ and $dy/dx = -2$.

C.F

$m^2 + 4m + 5 = 0$

This is a complex solution

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm \sqrt{-1}\sqrt{4}}{2}$

$= \frac{-4 \pm j2}{2}$

$= -2 \pm j.$

$m_1 = -2 + j \quad m_2 = -2 - j$

$y = e^{-2x}(A\cos x + B\sin x).$

P.I.

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x})$
$= 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 5C = 2.$

$C = 2/1.$

$y = 2e^{-2x}$

$G.S = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

at $x = 0$, $y = 1$

$1 = e^{-2(0)}(A\cos 0 + B\sin 0) + 2e^{-2(0)}$

$1 = 1(A) + 2$

$1 = A + 2$

$A = 1 - 2$

$A = -1$

$dy/dx = e^{-2x}[-A\sin x + B\cos x] + [A\cos x + B\sin x]$

$-2e^{-2x} + 2e^{-2x}$

$\frac{dy}{dx} = e^{-2x}[-A\sin x + B\cos x] - 2e^{-2x}(A\cos x + B\sin x) - 4e^{-2x}$

at $x = 0$ $dy/dx = -2$

$-2 = e^{-2(0)}(-A\sin 0 + B\cos 0) - 2e^{-2(0)}(A\cos 0 + B\sin 0) - 4e^{-2(0)}$

$-2 = B - 2A - 4$

$B = -2 - 2 + 4$

$B = 0$

$\therefore$ P.S $= e^{-2x}(-\cos x) + 2e^{-2x}$

$= -e^{-2x}\cos x + 2e^{-2x}$.

$= e^{-2x}(2 - \cos x)$

(7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

C.F

$3m^2 - 2m - 1 = 0$

$(3m+1)(m-1) = 0$

$3m = -1$ or $m = 1$

$m_1 = -\frac{1}{3}$ or $m_2 = 1$

$y = Ae^{-\frac{1}{3}x} + Be^{x}.$

P.I.

$y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$3(0) - 2(C) - [Cx+D]^2 = 2x - 3$

$0 - 2C - Cx - D = 2x - 3$

Comparing Coefficients.

x: $-C = 2$

$C = -2$

Constants: $-2C - D = -3$

~~Con~~tants: $4 - D = -3$

$D = 7$

$\Rightarrow y = -2x + 7$

G.S

$= Ae^{-\frac{1}{3}x} + Be^{x} - 2x + 7$

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Q8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

C.F

$m^2 - 6m + 8 = 0$

$(m-4)(m-2) = 0$

$m_1 = 4 \quad m_2 = 2$.

$y = Ae^{4x} + Be^{2x}$

P.I.

$y = Cxe^{4x}$.

$\frac{dy}{dx} = [x \cdot 4e^{4x} + e^{4x}]c$

$\frac{d^2y}{dx^2} = 4c[x4e^{4x} + e^{4x}] + 4ce^{4x}$.

$= 16cxe^{4x} + 4ce^{4x} + 4ce^{4x}$

$\Rightarrow 16cxe^{4x} + 8ce^{4x} - 6[4cxe^{4x} + ce^{4x}] + 8cxe^{2x} = 8e^{4x}$.

$16cxe^{4x} + \cancel{4ce^{4x}} + \cancel{4ce^{4x}} - 24cxe^{4x} - 6ce^{4x} + \cancel{8cxe^{4x}}$

$= 8e^{4x}$

$16cx + 4c + 4c - 24cx - 6c + 8cx = 8$

$= -8cx + 2c + 8cx = 8$.

$2c = 8$

$c = 8/2$

$c = 4$ //

$\therefore y = 4xe^{4x}$.

$G.S = C.F + P.I$

$= Ae^{4x} + Be^{2x} + 4xe^{4x}$.