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15/ENG04/003

ELECTRICAL ELECTRONICS

ENG 381 ASSIGNMENT

$$1) \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

C.F

$$m^2 - m - 2 = 0$$

$$(m-2)(m+1) = 0$$

$$m = 2 \text{ or } m = -1$$

$$y = Ae^{m_1x} + Be^{m_2x}$$

$$y = Ae^{2x} + Be^{-x}$$

P.I

$$y = C$$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

$$= 0 - 0 - 2C = 8$$

$$2C = -8$$

$$C = \frac{-8}{2}$$

$$C = -4$$

The General solution

$$G.S = C.F + P.I$$

$$= Ae^{2x} + Be^{-x} - 4$$

$$2) \frac{d^2 y}{dx^2} - 4y = 10e^{3x}$$

C.F

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \sqrt{4}$$

$$m = \pm 2$$

$$y = A \cosh 2x + B \sinh 2x$$

P.I

$$y = Ce^{3x}$$

$$\frac{dy}{dx} = 3Ce^{3x}$$

$$\frac{d^2 y}{dx^2} = 9Ce^{3x}$$

$$= 9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

Dividing through by e^{3x}

$$9C - 4C = 10$$

$$5C = 10$$

$$C = 2.$$

∴ The general equation becomes

C.F + P.I

$$i.e = A \cosh 2x + B \sinh 2x + 2e^{3x}$$

$$3) \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

C.F

$$m^2 + 2m + 1 = 0$$

$$(m+1)(m+1) = 0$$

$$m = -1 \text{ or } m = -1$$

$$y = e^{-x}(A+Bx)$$

P.I

$$y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$Ce^{-2x} = e^{-2x}$$

$$C = 1$$

The general solution will be
 $= e^{-x}(A+Bx) + e^{-2x}$

$$4) \frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

C.F

$$m^2 + 25 = 0$$

$$m^2 = -25$$

$$m = \sqrt{-25}$$

$$= \sqrt{-1} \sqrt{25}$$

$$m = j5$$

$$y = A \cos 5x + B \sin 5x$$

$$= A \cos 5x + B \sin 5x$$

P.I

$$y = Cx^2 + Dx + E$$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$$= 2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$= 2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$$

Comparing Coefficient

$$25C = 5$$

$$C = \frac{1}{5}$$

$$25D = 1$$

$$D = \frac{1}{25}$$

$$2C + 25E = 0$$

$$2\left(\frac{1}{5}\right) + 25E = 0$$

$$25E = -\frac{2}{5}$$

$$E = -\frac{2}{125}$$

∴ The general solution becomes
C.F + P.I

$$i.e. A(\cos 5x + B\sin 5x + \frac{25x^2 + 5x - 2}{125})$$

$$5) \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

C.F

$$m^2 - 2m + 1 = 0$$

$$(m-1)(m-1) = 0$$

$$m = 1 \text{ or } m = 1$$

$$y = e^x(A + Bx)$$

P.I

$$y = C\cos x + D\sin x$$

$$\frac{dy}{dx} = -C\sin x + D\cos x$$

$$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$$

$$-C\cos x - D\sin x = -2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$$

$$* C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$$

$$2C\sin x - 2D\cos x = 4\sin x$$

Comparing the coefficient

$$2C = 4$$

$$C = 4/2 \therefore C = 2$$

$$2D = 0$$

$$D = 0$$

$$P.I \quad y = 2\cos x + D\sin x = 2\cos x$$

General solution becomes

$$e^x(A + Bx) + \underline{\underline{2\cos x}}$$

$$26) \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

given that at $x=0$, $y=1$ and $\frac{dy}{dx} = -2$

C.F

$$m^2 + 4m + 5 = 0$$

This is a complex situation

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm j2}{2}$$

$$= -2 \pm j$$

$$m_1 = -2 + j \quad m_2 = -2 - j$$

$$y = e^{-2x} (A \cos x + B \sin x)$$

P.I

$$y = ce^{-2x}$$

$$\frac{dy}{dx} = -2ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4ce^{-2x}$$

$$4ce^{-2x} + 4(-2ce^{-2x}) + 5(ce^{-2x}) = 2e^{-2x}$$

$$4ce^{-2x} - 8ce^{-2x} + 5ce^{-2x} = 2e^{-2x}$$

$$4c - 8c + 5c = 2$$

$$c = 2$$

$$y = 2e^{-2x}$$

$$G.S = e^{-2x} (A \cos x + B \sin x) + 2e^{-2x} \text{ at } x=0, y=1$$

$$1 = e^{-2 \cdot 0} (A \cos 0 + B \sin 0) + 2e^{-2 \cdot 0}$$

$$1 = 1(A) + 2$$

$$1 = A + 2$$

$$A = 1 - 2$$

$$A = -1$$

$$\frac{dy}{dx} = e^{-2x} [-A \sin x + B \cos x] + [A \cos x + B \sin x] - 2e^{-2x} + 2e^{-2x}$$

$$\frac{dy}{dx} = e^{-2x} [-A \sin x + B \cos x] - 2e^{-2x} (A \cos x + B \sin x) - 4e^{-2x}$$

$$\text{at } x=0 \quad \frac{dy}{dx} = -2$$

$$-2 = e^{-2(0)} (-A \sin 0 + B \cos 0) - 2e^{-2(0)} (A \cos 0 + B \sin 0) - 4e^{-2(0)}$$

$$-2 = B - 2A - 4$$

$$B = -2 - 2A + 4$$

$$B = 0$$

$$\therefore P.S = e^{-2x} (-\cos x) + 2e^{-2x}$$

$$= -e^{-2x} \cos x + 2e^{-2x}$$

$$= e^{-2x} (2 - \cos x)$$

$$7) \quad 3 \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - y = 2x^{-3}$$

C.F

$$3m^2 - 2m - 1 = 0$$

$$(3m+1)(m-1) = 0$$

$$3m = -1 \text{ or } m = 1$$

$$m = -\frac{1}{3} \text{ or } m = 1$$

$$y = A e^{-1/3x} + B e^x$$

P.I

$$y = Cx + D$$

$$\frac{dy}{dx} = C$$

$$\frac{d^2y}{dx^2} = 0$$

$$3(0) - 2(0) - [Cx + D]^2 = 2x - 3$$

$$0 - 2C - (x + D) = 2x - 3$$

Comparing Coefficients

$$2x : -C = 2$$

$$C = -2$$

$$\text{Constants: } -2C - D = -3$$

$$\text{Ans: } 4 - D = -3$$

$$D = 7$$

$$= y = -2x + 7$$

C.S

$$= Ae^{-1/3x} + Be^x - 2x + 7$$

$$8) \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

C.F

$$m^2 - 6m + 8 = 0$$

$$(m-4)(m-2) = 0$$

$$m_1 = 4 \quad m_2 = 2$$

$$y = Ae^{4x} + Be^{2x}$$

P.I

$$y = Cx e^{4x}$$

$$\frac{dy}{dx} = [x \cdot 4e^{4x} + e^{4x}] C$$

$$\frac{d^2y}{dx^2} = 4C[x + 1]e^{4x} + 4Ce^{4x}$$

$$= 16Cx e^{4x} + 4Ce^{4x} + 4Ce^{4x}$$

$$16Cx e^{4x} + 8Ce^{4x} - 6[4Cx e^{4x} + e^{4x}] + 8Cx e^{2x} = 8e^{4x}$$

$$16Cx e^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cx e^{4x} - 6e^{4x} + 8Cx e^{2x} = 8e^{4x}$$

$$16Cx + 4C + 4C - 24Cx - 6C + 8C = 8$$

$$-8Cx + 2C + 8Cx = 8$$

$$2C = 8$$

$$C = 4$$

$$C = 4$$

$$y = 4x e^{4x}$$

$$\text{C.S} = \text{C.F} + \text{P.I}$$

$$= Ae^{4x} + Be^{2x} + 4x e^{4x}$$