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15/ENG05/006
MECHATRONICS ENGR

ENG 381 ASSIGNMENT I

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$ ——— (*)

let $y = e^{kx}$; $y' = ke^{kx}$; $y'' = k^2e^{kx}$

$k^2e^{kx} - ke^{kx} - 2e^{kx} = 8$

$(k^2 - k - 2)e^{kx} = 0$ (using the L.H.S as the homogenous term)

since $e^{kx} = y$, then, $k^2 - k - 2 = 0$

$k^2 - 2k + k - 2 = 0$

$k(k-2) + 1(k-2) = 0$

$(k+1)(k-2) = 0$

$k_1 = -1$ and $k_2 = 2$

$y_1 = e^{k_1x} = e^{-x}$

$y_2 = e^{k_2x} = e^{2x}$

$y = C_1y_1 + C_2y_2$

$y = C_1e^{-x} + C_2e^{2x}$

For the non-homogenous part, y_p:

$y = A$; $y' = 0$; $y'' = 0$

substituting into eqn (*)

$0 - 0 - 2A = 8$

$-2A = 8$

$A = -8/2 = -4$

$\therefore y = y_h + y_p$

$y = C_1e^{-x} + C_2e^{2x} - 4$

2.) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Let $y = e^{kx}$; $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$k^2e^{kx} - 4e^{kx} = 10e^{3x}$

$(k^2-4)e^{kx} = 0$ (solving as a homogenous part)

$k^2 - 4 = 0$

$k = \pm 2$

$y_1 = e^{k_1x} = e^{2x}$

$y_2 = e^{k_2x} = e^{-2x}$

$y_h = C_1e^{2x} + \cancel{C_2e^{-2x}} \Rightarrow C_1\cosh 2x + C_2\sinh 2x$

For the non-homogenous part, y_p:

$y = Ce^{3x}$

$y' = 3Ce^{3x}$; $y'' = 9Ce^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9C - 4C = 10$

$5C = 10$

$C = 2$

$\therefore$ The general solution is $C_1\cosh 2x + C_2\sinh 2x + 2e^{3x}$

3.) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

solving as a homogenous part,

$y = e^{kx}$; $y' = ke^{kx}$; $y'' = k^2e^{kx}$

$k^2e^{kx} + 2ke^{kx} + e^{kx} = e^{-2x}$

$(k^2 + 2k + 1)e^{kx} = 0$

$k^2 + 2k + 1 = 0$

$k^2 + k + k + 1 = 0$

$k(k+1) + 1(k+1) = 0$

$(k+1)^2 = 0$

$k_1 = k_2 = -1$

$y_h = C_1e^{-x} + C_2xe^{-x}$

For the non-homogenous part,

$y = Ce^{-2x}$; $y' = -2Ce^{-2x}$

$y'' = 4Ce^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$4C - 4C + C = 1$

$C = 1$

General solution:

$y = C_1e^{-x} + C_2xe^{-x} + e^{-2x}$

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4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

for the homogenous part $y = e^{kx}$

$k^2 + 25 = 0$

$k^2 = -25$

$k = \sqrt{-25}$

$k = 5j$

$y = A\cos nx + B\sin nx$

$= A\cos 5x + B\sin 5x$

for the non-homogenous part,

$y = Cx^2 + Dx + E$

$y' = 2Cx + D$

$y'' = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

comparing coefficients

$25C = 5$

$C = 1/5$

$25D = 1$

$D = 1/25$

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$25E = -2/5$

$E = -2/125$

The general solution:

$A\cos 5x + B\sin 5x + \frac{25x^2 + 5x - 2}{125}$

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5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$ —— (*)

let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

solving L.H.S as a homogenous term,

$k^2e^{kx} - 2ke^{kx} + e^{kx} = 0$

$(k^2 - 2k + 1)e^{kx} = 0$

since $y = e^{kx}$, then $k^2 - 2k + 1 = 0$

$k^2 - k - k + 1 = 0$

$k(k-1) - 1(k-1) = 0$

$(k-1)^2 = 0$

$k = k_1 = k_2 = 1$

$y_1 = e^{k_1x} = e^x$

$y_2 = e^{k_2x} = e^x$

$y_h = C_1y_1 + xC_2y_2$

$y_h = C_1e^x + xC_2e^x$

For non-homogenous part, y_p:

$y = A\sin x + B\cos x$

$y' = A\cos x - B\sin x$

$y'' = -A\sin x - B\cos x$

substituting into eqn (*)

$-A\sin x - B\cos x - 2(A\cos x - B\sin x) + A\sin x + B\cos x = 4\sin x$

$-A\sin x - B\cos x - 2A\cos x + 2B\sin x + A\sin x + B\cos x = 4\sin x$

$2B\sin x - 2A\cos x = 4\sin x$

$-2A = 0$; $A = 0$

$2B = 4$; $B = 2$

$\therefore y = C_1e^x + C_2xe^x + 2\sin x$

No 6)

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

given that at $x=0$, $y=1$

and $\frac{dy}{dx} = -2$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16-20}}{2}$

$\frac{-4 \pm \sqrt{-4}}{2}$

$\frac{-4 \pm \sqrt{-1} \times \sqrt{4}}{2}$

$= \frac{-4 \pm j2}{2}$

$= -2 \pm j$

$m = -2+j$ and $m_2 = -2-j$

$y = e^{-2x}(A\cos x + B\sin x)$

P.I

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5Ce^{-2x} = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 5C = 2$

$C = 2$

$y = 2e^{-2x}$

General solution

$= e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

at $x=0$, $y=1$

(7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x-3$

C.I

$3m^2 - 2m - 1 = 0$

$(3m+1)(m-1) = 0$

$3m = -1$ or $m = 1$

$m = -\frac{1}{3}$, $m = 1$

$y = Ae^{-\frac{1}{3}x} + Be^{x}$

P.I

$y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$3(0) - 2(C) - [(Cx+D)] = 2x-3$

$0 - 2C - Cx - D = 2x - 3$

Comparing coefficient

$x^0 - C = 2$

$C = -2$

Constants $\begin{cases} -2C - D = -3 \\ 4 - D = -3 \end{cases}$

$D = 7$

$y = -2x + 7$

General solution

$Ae^{-\frac{1}{3}x} + Be^{x} - 2x + 7$

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8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

for the homogenous part,

$k^2 - 6k + 8 = 0$

$(k-4)(k-2) = 0$

$k_1 = 4,\ k_2 = 2$

$y = C_1 e^{4x} + C_2 e^{2x}$

for the non-homogenous part,

$y = Cxe^{4x}$

$\frac{dy}{dx} = (x \cdot 4e^{4x} + e^{4x})C$

$\frac{d^2y}{dx^2} = 4C[x\,4e^{4x} + e^{4x}] + 4Ce^{4x}$

$= 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$16Cxe^{4x} + 8Ce^{4x} - 6[4Cxe^{4x} + Ce^{4x}] + 8Cxe^{2x} = 8e^{4x}$

$16Cx + 4C + 4C - 24Cx - 6C + 8Cx = 8$

$-8Cx + 2C + 8Cx = 8$

$2C = 8$

$C = 8/2$

$C = 4$

$y_p = 4xe^{4x}$

$y = Ae^{4x} + Be^{2x} + 4xe^{4x}$

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