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15/ENG01/020
CHEMICAL ENGINEERING

① $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2 = 0$

$(m-2)(m+1) = 0$

$m = 2, \quad m = -1$

$y = Ae^{m_1 x} + Be^{m_2 x}$

$y = Ae^{2x} + Be^{-x}$

P.I

$y = c$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$= 0 - 0 - 2c = 8$

$c = -8/2$

$c = -4$

The general Solution =

$G.S = C.F + P.I$

$= Ae^{2x} + Be^{-x} - 4$

② $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

C.F

$m^2 - 4 = 0$

$m^2 = 4$

$m = \sqrt{4}$

$m = \pm 2$

$y = A\cosh 2x + B\sinh 2x$

P.I

$y = ce^{3x}$

$\frac{dy}{dx} = 3ce^{3x}$

$\frac{d^2y}{dx^2} = 9ce^{3x}$

$9ce^{3x} - 4(ce^{3x}) = 10e^{3x}$

$9ce^{3x} - 4ce^{3x} = 10e^{3x}$

Divide through by e^{3x}

$9c - 4c = 10$

$5c = 10$

$c = 2$

$G.S = CF + PI$

$= A\cosh 2x + B\sinh 2x + 2e^{3x}$

③ $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

C.F

$m^2 + 2m + 1 = 0$

$(m+1)(m+1) = 0$

$m_1 = -1, \quad m_2 = -1$

$m = -1$ twice

$y = e^{-x}(A + Bx)$

P.I

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1$

The general solution will be

$= e^{-x}(A + Bx) + e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

C·F

$m^2 + 25 = 0$

$m^2 = -25$

$m = \sqrt{-25}$

$= \sqrt{-1}\,\sqrt{25}$

$m = j5$

$y = A\cos nx + B\sin nx$

$= A\cos 5x + B\sin 5x$

P·I

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$= 2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$= 2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

Comparing Coefficient

$25C = 5$

$C = 1/5$

$25D = 1$

$D = 1/25$

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$25E = -2/5$

$E = \frac{-2}{125}$

∴ The general solution becomes

CF + P·I

i.e $A\cos 5x + B\sin 5x + \frac{25x^2 + 5x - 2}{125}$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

C·F

$m^2 - 2m + 1 = 0$

$(m-1)(m-1) = 0$

$m = 1$ or $m = 1$

$y = e^{x}(A + Bx)$

P·I

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$-C\cos x - D\sin x = -2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$2C\sin x - 2D\cos x = 4\sin x$

Comparing the coefficient

$2C = 4$

$C = 4/2$; $C = 2$

$2D = 0$

$D = 0$

$P.I$ $y = 2\cos x + 0\sin x = 2\cos x$

General solution becomes

$e^{x}(A+Bx) + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

Given that at $x=0$, $y=1$ and $\frac{dy}{dx} = -2$

C.F

$m^2 + 4m + 5 = 0$

The equation above is a complex solution

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16-20}}{2}$

$= \frac{-4 \pm \sqrt{-1}\sqrt{4}}{2}$

$= \frac{-4 \pm j2}{2}$

$= -2 \pm j$

$m_1 = -2 + j \quad m_2 = -2 - j$

$y = e^{-2x}(A\cos x + B\sin x)$

P.I

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 5C = 2$

$C = 2$

$y = 2e^{-2x}$

$G.S = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

at $x=0$, $y=1$

$1 = e^{-0}(A\cos 0 + B\sin 0) + 2e^{-2(0)}$

$1 = 1(A) + 2$

$1 + A + 2$

$A = 1 - 2$

$A = -1$

$dy/dx = e^{-2x}[-A\sin x + B\cos x] + [A\cos x + B\sin x]$

$-2e^{-2x} + 2e^{-2x}$

$dy/dx = e^{-2x}[-A\sin x + B\cos x]$
$2e^{-2x}(A\cos x + B\sin x) - 4e^{-2x}$

at $x=0$ $dy/dx = -2$

$-2 = e^{-2(0)}(-A\sin 0 + B\cos 0) - 2e^{-2(0)}(A\cos 0 + B\sin 0) - 4e^{-2(0)}$

$-2 = B - 2A - 4$

$B = -2 - 2 + 4$

$B = 0$

$\therefore P.S = e^{-2x}(-\cos x) + 2e^{-2x}$

$= -e^{-2x}(\cos x + 2e^{-2x}$

$= e^{-2x}(2 - \cos x)$

7) $3\,d^2y/dx^2 - 2\,dy/dx - y = 2x^2$

C.F

$3m^2 - 2m - 1 = 0$

$(3m+1)(m-1) = 0$

$3m = -1$ or $m = 1$

$m = -1/3$ or $m = 1$

$y = Ae^{-x/3} + Be^{x}$

P.I

$y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$3(0) - 2(C) - [(Cx + D)] = 2x - 3$

$0 - 2C - Cx - D = 2x - 3$

Comparing coefficients

$x: -C = 2$

$C = -2$

Constants: $-2C - D = -3$

$4 - D = -3$

$D = 7$

$y = -2x + 7$

$= Ae^{-1/3x} + Be^{x} - 2x + 7$

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

C.F

$m^2 - 6m + 8 = 0$

$(m-4)(m-2) = 0$

$m_1 = 4 \quad m_2 = 2$

$y = Ae^{4x} + Be^{2x}$

P.I

$y = Cxe^{4x}$

$\frac{dy}{dx} = [x \cdot 4e^{4x} + e^{4x}]C$

$\frac{d^2y}{dx^2} = 4C[x4e^{4x} + e^{4x}] + 4Ce^{4x}$

$= 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$16Cxe^{4x} + 8Ce^{4x} - 6[4Cxe^{4x} + Ce^{4x}]$

$+ 8Cxe^{4x} = 8e^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x}$

$- 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$16Cx + 4C + 4C - 24Cx - 6C$

$+ 8Cx = 8$

$-8Cx + 2C + 8Cx = 8$

$2C = 8$

$C = 8/2$

$C = 4$

$y = 4xe^{4x}$

$G.S = CF + PI$

$= Ae^{4x} + Be^{2x} + 4xe^{4x}$