Cony-Tye Afoun
KS/eng/01/073

Assignment

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2 = 0$

$m_1 = 2, m_2 = -1$

$y = Ae^{2x} + Be^{-x}$

$f(x) = 8$

$y = C$

$\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2c = 8$

$\frac{-2c}{-2} = \frac{8}{-2}$

$c = -4$

$y = Ae^{2x} + Be^{-x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$m = \pm\sqrt{4}$

$m = \pm 2$

$y = Ae^{2x} + Be^{-2x}$

$y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$9C - 4C = 10$

$\frac{5C}{5} = \frac{10}{5}$

$C = 2$

$y = 2e^{3x}$

$y = Ae^{2x} + Be^{-2x} + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$m^2 + 2m + 1 = 0$

$m = m_1 = m_2$

$m = 1$

$y = Ae^{-x} + Be^{-x}$

$y = e^{-x}(A + Bx)$

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2e^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$4C - 4C + C = 1$

$C = 1$

$y = e^{-2x}$

$y = e^{-x}(A + Bx) + e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + 4$

$m^2 + 25 = 0$

$m = \pm\sqrt{25}$

$m = \pm 5$

$y = C\cos 5x + D\sin 5x$

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + 4$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + 4$

$\frac{25Cx^2}{25} = \frac{5x^2}{25}$

$C = 1/25$

$25B x = x$

$B = 1/25$

$2C + 25E = 0$

$2(1/25) + 25E = 0$

$25E = -2/5$

$E = \frac{-2}{125}$

$y = Cx^2 + Bx + E = 1/B x^2 + 1/25 x - \frac{2}{125}$

$y = C\cos 5x + B\sin 5x + 1/125(25x^2 + 5x + 2)$

(5) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$m = m_1 = m_2$

$m = 1$

$y = Ae^{mx} + Be^{m2x}$

$y = e^{mx}(A + Bx)$

$y = e^x(A + Bx)$

$f(x) = 4\sin x$

$y = C\cos x + B\sin x$

$\frac{dy}{dx} = -C\sin x + B\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - B\sin x$

$-C\cos x - B\sin x - 2(-C\sin x + B\cos x)$

$+ C\cos x + B\sin x = 4\sin x$

$-C\cos x - B\sin x + 2C\sin x - 2B\cos x$

$+C\cos x + B\sin x = 4\sin x$

$-C\cos x - 2B\cos x + C\cos x - B\sin x$

$+ 2C\sin x + B\sin x = 4\sin x$

$\cos x(-C - 2B + C) + \sin x(-B + 2C + B) = 4\sin x$

$= 4\sin x$

$\cos x(-2B) + \sin x(2C) = 4\sin x$

$-2B = 0$ $\quad 2C = 4$

$B = 0$ $\quad C = 4/2$

$C = 2$

$y = 2\cos x + B\sin x$

$y = 2\cos x$

$y = e^x(A + Bx) + 2\cos x$

(6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$x = 0, y = 1$ and $\frac{dy}{dx} = -2$

$m^2 + 4m + 5 = 0$

$m = -2 \pm j$

$y = e^{-2x}(C\cos x + B\sin x)$

$y = Cxe^{-2x}$

$\frac{dy}{dx} = Cx(-2e^{-2x}) + e^{-2x}(C)$

$= -2Cxe^{-2x} + Ce^{-2x}$

$\frac{d^2y}{dx^2} = -2Cx(-2e^{-2x}) + e^{-2x}(-2C)$

$+ 2Ce^{-2x}$

$= 4Cxe^{-2x} - 2Ce^{-2x} - 2Ce^{-2x}$

$= 4Cxe^{-2x} - 4Ce^{-2x}$

$4Cxe^{-2x} - 4Ce^{-2x} - 8Cxe^{-2x} +$

$4Ce^{-2x} + 5Cxe^{-2x}$

$= 2e^{-2x}$

$4Cxe^{-2x} - 8Cxe^{-2x} + 5Cxe^{-2x}$

$- 4Ce^{-2x} + 4Ce^{-2x} = 2e^{-2x}$

$e^{-2x}(4Cx - 8Cx + 5Cx) = 2e^{-2x}$

$Cx = 2$

$y = 2e^{-2x}$

$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$

at $x=0, y=1$

$y = e^{-2(0)}((C\cos(0) + D\sin(0)) + 2e^{-2(0)}$

$y = 1(C+0) + 2$

$y = C + 0 + 2$

$y = C + 2$

$C = -1$

$\frac{dy}{dx} = e^{-2x}(-C\sin x + D\cos x) + (C\cos x + D\sin x)$

$= 2e^{-2x} + 4xe^{-2x}$

at $x=0, y=1$ and $\frac{dy}{dx} = -2$

$-2 = D - 2C - 4$

$-2 + 4 = 2 - 2C$

$2 = D + 2$

$D = 0$

$y = e^{-2x}(-\cos x + 0) + 2e^{-2x}$

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

(7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$m_1 = 1, m_2 = -1/3$

$y = Ae^{x} + Be^{-x/3}$

$y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx} = 0$

$0 - 2C - Cx + D = 2x - 3$

$-Cx - 2C - D = 2x + 3$

$-C = 2$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$D = 7$

$y = -2x + 7$

$y = Ae^{x} + Be^{-x/3} - 2x + 7$

(8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$m_1 = 4, m_2 = 2$

$y = Ae^{4x} + Be^{2x}$

$y = Cxe^{4x}$

$\frac{dy}{dx} = Cx(4e^{4x}) + e^{4x}(C)$

$\frac{d^2y}{dx^2} = Cx(16e^{4x}) + 4e^{4x}(C) + Ce^{4x}$

$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Ce^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$e^{4x}(16Cx - 24Cx + 8Cx + 4C + 4C - 6C) = 8e^{4x}$

$e^{4x}(0 + 2C) = 8e^{4x}$

$\frac{2C}{2} = \frac{8}{2}$

$C = 4$

$y = 4xe^{4x}$

$y = Ae^{4x} + Be^{2x} + 4xe^{4x}$