1 $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , $m = \frac{+\cancel{b} \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$

$m = \frac{1 \pm \sqrt{9}}{2}$ $m_1 = 2$ $m_2 = -1$

$y = Ae^{m_1 x} + Be^{m_2 x}$

C.F: $y = Ae^{2x} + Be^{-x}$

PI: $y = c$

$\frac{dy}{dx} = 0$ , $\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2(c) = 8$

$-2c = 8$

$c = -4$

P.I: $y = -4$

G.S = C.F + P.I

G.S: $y = Ae^{2x} + Be^{-x} - 4$

2 $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m_1 = 2$, $m_2 = -2$

C.F: $y = A\cosh 2x + B\sinh 2x$

P.I: $y = ce^{3x}$

$\frac{dy}{dx} = 3ce^{3x}$ , $\frac{d^2y}{dx^2} = 9ce^{3x}$

$9ce^{3x} - 4(ce^{3x}) = 10e^{3x}$

$9c - 4c = 10$

$5c = 10$ , $c = 2e^{3x}$

P.I: $y = 2e^{3x}$

G.S: $y = A\cosh 2x + B\sinh 2x + 2e^{3x}$

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3. $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = e^{-2x}$

$m^2 + 2m + 1 = 0$

$(m+1)(m+1)$

$m = -1$ twice

C.F : $y = e^{-x}(A + Bx)$

P.I : $y = ce^{-2x}$

$\dfrac{dy}{dx} = -2ce^{-2x}$ , $\dfrac{d^2y}{dx^2} = 4ce^{-2x}$

$4ce^{-2x} + 2(-2ce^{-2x}) + ce^{-2x} = e^{-2x}$

$4c - 4c + c = 1$

$c = 1$

P.I : $y = 1e^{-2x}$

G.S : $y = e^{-x}(A + Bx) + e^{-2x}$

4 $\dfrac{d^2y}{dx^2} + 25y = 5x^2 + x$

$m^2 + 25 = 0$

$m^2 = -25$

$m = 5j$

C.F : $y = e^{0x}(A\cos 5x + B\sin 5x)$

P.I : $y = Cx^2 + Dx + E$

$\dfrac{dy}{dx} = 2Cx + D$ , $\dfrac{d^2y}{dx^2} = 2C$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$\cancel{2C + E = 0}$ $\qquad$ $2C + 25E = 0$

$\cancel{C = 5}$ $\qquad$ $C = \dfrac{5}{25} = \dfrac{1}{5}$

$\cancel{D = 1}$ $\qquad$ $D = \dfrac{1}{25}$

$\cancel{E = -2(5) \,,\, E = -10}$ $\qquad$ $E = -\dfrac{2}{5} \times \dfrac{1}{25}$

$\cancel{\text{P.I : } y = 5x^2 + x - 10}$ $\qquad$ $E = -\dfrac{2}{125}$

$\cancel{\text{G.S : } y = e^{0x}(A\cos 5x + B\sin 5x) + 5x^2 + x - 10}$

P.I : $\dfrac{1}{5}x^2 + \dfrac{1}{25}x - \dfrac{2}{125} = y$

G.S : $y = e^{0x}(A\cos 5x + B\sin 5x) + \dfrac{1}{5}x^2 + \dfrac{1}{25}x - \dfrac{2}{125}$

$y = A\cos 5x + B\sin 5x + (x^2 + \frac{1}{5}x - \frac{2}{25})\frac{1}{5}$

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5 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$(x-1)(x-1)$

$x = 1$ Twice

C.F: $y = e^{x}(A + Bx)$

P.I: $y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$ , $\frac{d^2y}{dx} = -C\cos x - D\sin x$

$-C\cos x - D\sin x + 2C\sin x + 2D\cos x + C\cos x + D\sin x = 4\sin x$

$\cos x(-C - 2D + C) + \sin x(-D + 2C + D) = 4\sin x$

$-2D = 0$

$2C = 4$ , $C = 2$ P.I: $y = 2\cos x + 0$

G.S: $y = e^{x}(A + Bx) + 2\cos x$

6 $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

given That $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , $m = \frac{-4 \pm \sqrt{16 - 4(1)(5)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$ , $m = \frac{-4 \pm 2j}{2}$ $m = -2 \pm j$

$\alpha = -2$ , $\beta = 1$

C.F: $y = e^{-2x}(A\cos 1x + B\sin 1x)$

P.I: $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$ , $\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$\cancel{4Ce^{-2x} - 10Ce^{-2x} + Ce^{-2x} = 2e^{-2x}}$

$\cancel{4C - 2C + C = 2}$

$\cancel{C = \frac{2}{3}}$

~~P.I: $= \frac{2}{3}e^{-2x}$~~

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2$

$C = 2$

P.I: $= 2e^{-2x}$

~~G.S: $y = e^{-2x}(A\cos 1x + B\sin 1x) + \frac{2}{3}e^{-3x}$~~

G.S: $y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

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$\frac{dy}{dx} = e^{-2x}(-A\sin 2x + B\cos 2x) + (-2e^{-4x}(A\cos 2x + B\sin 2x)) + (-\frac{4}{3}e^{-2x})$

$\frac{dy}{dx} = e^{-2x}(-A\sin 2x + B\cos 2x) - 2e^{-2x}(A\cos 2x + B\sin 2x) - \frac{4}{3}e^{-2x}$

if $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$

$1 = e^{-2(0)}(A\cos 2(0) + B\sin 2(0)) + \frac{2}{3}e^{-2(0)}$

$1 = 1(A) + \frac{2}{3}$

$A = \frac{1}{3}$ $\quad A = -1$

$-2 = e^{-2(0)}(-A\sin 2(0) + B\cos 2(0)) - 2e^{-2(0)}(A\cos 0 + B\sin 0) - \frac{4}{3}e^{-0}$

$-2 = 1(B) - 1(A) - \frac{4}{3}$ $\quad -2 = B - 1 - 4$

$B = 5$

$-2 = B - \frac{1}{3} - \frac{4}{3}$

$B = \frac{5}{3} - 2$

$B = -\frac{1}{3}$

G.S: $y = e^{-2x}(-1\cos 2x - 5\sin 2x) + \frac{2}{3}e^{-2x}$

$\cancel{y = \frac{1}{3}e^{-4x}(\cos 2x - \sin 2x) + \frac{2}{3}e^{-2x}}$

7 $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$(m - 1)(3m + 1)$

$m_1 = 1$, $m_2 = -\frac{1}{3}$

C.F: $y = Ae^{x} + Be^{-x/3}$

P.I: $y = Cx + D$

$\frac{dy}{dx} = C$, $\frac{d^2y}{dx^2} = 0$

$3(0) - 2(C) - Cx + D = 2x - 3$

$-2C - Cx - D = 2x - 3$

$-Cx = 2x$, $C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = 3$

$D = 7$

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P.I: $y = -2x + 7$

G.S: $y = Ae^{x} + Be^{-x/3} - 2x + 7$

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$(m-4)(m-2)$

$m_1 = 4, \quad m_2 = 2$

C.F: $y = Ae^{4x} + Be^{2x}$

P.I: $y = ce^{4x}$

$\frac{dy}{dx} = 4ce^{4x}, \quad \frac{d^2y}{dx^2} = 16ce^{4x}$

$16ce^{4x} - 6(4ce^{4x}) + 8(ce^{4x}) = 8e^{4x}$

$16c - 24c + 8c = 8\cancel{e}$

$0c = 8e$

$c = 0$

G.S: $y = Ae^{4x} + Be^{2x}$

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