

Maths Assignment

$$1.) \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

$$m^2 - m - 2 = 0$$

$$m^2 + m - 2m - 2 = 0$$

$$m(m+1) - 2(m+1) = 0$$

$$m+1 = 0 \quad m-2 = 0$$

$$m = -1 \quad m = 2$$

$$C.F = y = Ae^{-2x} + Be^{2x}$$

Particular Integral

$$y = C$$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

$$0 - 0 - 2C = 8$$

$$-2C = 8$$

$$y = C = -4$$

$$\bullet \text{ G.E} = C.F + P.I$$

$$y = Ae^{-2x} + Be^{2x} - 4$$

$$2.) \frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

solution

$$m^2 - 2^2 = 0$$

$$m^2 = 2^2$$

$$m = \pm 2$$

$$C.F. = y = C \cosh 2x + D \sinh 2x$$

$$P.I. \quad y = Ce^{3x}$$

$$\frac{dy}{dx} = 3Ce^{3x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

$$e^{3x}(9C - 4C) = 10e^{3x}$$

$$\frac{e^{3x}(5C)}{e^{3x}} = \frac{10e^{3x}}{e^{3x}}$$

$$5C = 10$$

$$C = 2$$

$$P.I. \quad y = 2e^{3x}$$

$$G.E = C \cosh 2x + D \sinh 2x + 2e^{3x}$$

$$3.) \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = e^{-2x}$$

solution

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$m+1 = 0$$

$$m+1 = 0$$

$$m = -1$$

$$m = -1$$

$$C.F = y = e^{-x}(A+Bx)$$

$$P.I \quad y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2 y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$e^{-2x}(4C - 4C + C) = e^{-2x}$$

$$e^{-2x}(C) = e^{-2x}$$

$$C = 1$$

$$P.I = e^{-2x}$$

$$G.E = y = e^{-x}(A+Bx) + e^{-2x}$$

$$4.) \frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$m^2 + 5^2 = 0$$

$$m^2 = -5^2$$

$$m = \pm \sqrt{-5^2}$$

$$m = \pm \sqrt{-1} \cdot \sqrt{5^2}$$

$$m = \pm j5$$

$$C.F = y = C \cos 5x + D \sin 5x$$

$$P.I = y = Cx^2 + Dx + E$$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$$= 2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$$

$$\cancel{(2/5)x^2 + 25} / \cancel{25} / \cancel{Dx} / \cancel{25}$$

$$= 25Cx^2 + 25Dx + 25E + 2C = 5x^2 + x$$

$$25C = 5$$

$$C = \frac{1}{5}$$

$$25D = 1$$

$$D = \frac{1}{25}$$

$$25E + 2C = 0$$

$$25E + \frac{2}{5} = 0$$

$$25E = -\frac{2}{5}$$

$$E = \frac{-2}{125}$$

$$P.I = y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$

$$G.E = C \cos 5x + D \sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$

$$5.) \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

Solution

$$m^2 - 2m + 1 = 0$$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - 1(m-1) = 0$$

$$m-1 = 0$$

$$m = 1$$

$$C.F = y = e^x (A + Bx)$$

$$P.I = y = C \cos x + D \sin x$$

$$\frac{dy}{dx} = -C \sin x + D \cos x$$

$$\frac{d^2y}{dx^2} = -C \cos x - D \sin x$$

$$-C \cos x - D \sin x - 2(-C \sin x + D \cos x) + C \cos x + D \sin x = 4 \sin x$$

$$\cos x (-C + 2D + C) + \sin x (-D - 2C + D) = 4 \sin x$$

$$\cos x (2D) + \sin x (-2C) = 4 \sin x$$

$$2D = 0$$

$$D = 0$$

$$-2C = 4$$

$$C = -2$$

$$P.I = y = -2\cos x$$

$$G.E = y = e^x(A+Bx) + -2\cos x$$

$$6.) \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

Solution

$$m^2 + 4m + 5 = 0$$

$$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$m = \frac{-4 \pm \sqrt{-4}}{2}$$

$$m = \frac{-4 \pm 2j}{2}$$

$$m = -2 \pm j$$

$$C.F = y = e^{-2x}(C\cos x + D\sin x)$$

$$P.I = y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} + -8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$$

$$e^{-2x}(4C - 8C + 5C) = 2e^{-2x}$$

$$e^{-2x}(C) = 2e^{-2x}$$

$$C = 2 \quad P.I = 2e^{-2x}$$

$$G.E = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$$

$$1 = e^{-2(0)} (C \cos(0) + D \sin(0)) + 2e^{-2(0)}$$

$$1 = C + 2$$

$$1 - 2 = C$$

$$C = -1$$

$$-2 = e^{-2(x)} (-C \sin(x) + D \cos(x)) + (C \cos(x) + D \sin(x)) \cdot (-2e^{2(x)}) + 2e^{2(x)}$$

$$-2 = D - 2C + 2$$

$$-4 = D - 2C$$

$$-4 = D - 2(-1)$$

$$-4 = D + 2$$

$$D = -6$$

$$P.S = y = e^{-2x} (-\cos x - 6 \sin x) + 2e^{-2x}$$

$$7.) \quad 3 \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} - y = 2x - 3$$

$$3m^2 - 2m - 1 = 0$$

$$m^2 - \frac{2}{3}m - \frac{1}{3} = 0$$

$$m^2 - m + \frac{1}{3}m - \frac{1}{3} = 0$$

$$m(m-1) + \frac{1}{3}(m-1) = 0$$

$$m-1 = 0 \quad m + \frac{1}{3} = 0$$

$$m = 1$$

$$m = -\frac{1}{3} = -0.33$$

$$CF = y = Ae^x + Be^{-0.33x}$$

$$P.I = y = Cx + D$$

$$\frac{dy}{dx} = C$$

$$\frac{dy}{dx^2} = 0$$

$$\textcircled{+} 0 - 2C - Cx - D = 2x - 3$$

$$\textcircled{+} \textcircled{-} (2-x) - D =$$

$$-C(2-x) - 2C - D = 2x - 3$$

$$-C(2-x) = 2x - 3$$

$$C = -2$$

$$-2C - D = -3$$

$$-2(-2) - D = -3$$

$$4 - D = -3$$

$$D = 7$$

$$P.I = y = -2x + 7$$

$$G.T = y = Ae^{3x} + Be^{-0.33x} - 2x + 7$$

$$8.) \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

solution

$$m^2 - 6m + 8 = 0$$

$$m^2 - 4m - 2m + 8 = 0$$

$$m(m-4) - 2(m-4) = 0$$

$$m-4=0$$

$$m-2=0$$

$$m=4$$

$$m=2$$

$$C.F = y = Ae^{4x} + Be^{2x}$$

$$P.I = y = Cx e^{4x}$$

$$\frac{dy}{dx} = 4Cx e^{4x} + C e^{4x}$$

$$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$$

$$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cx e^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$$

$$Ce^{4x} (16x + 4 + 4 - 24x - 6 + 8x) = 8e^{4x}$$

$$Ce^{4x} (2) = 8e^{4x}$$

$$2C = 8$$

$$C = 4$$

$$P.I = y = 4x e^{4x}$$

$$A.S = y = Ae^{4x} + Be^{2x} + 4x e^{4x}$$