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MECHANICAL ENGINEERING

ENG 281 ASSIGNMENT TWO

ENGINEERING MATHEMATICS

The parametric equations of a curve are as given on equations (1) and (2):

$x = \cos t + t \sin t$ —— (1)

$y = \sin t - t \cos t$ —— (2)

In terms of t determine:

i) an expression for the radius of curvature (R), and

ii) expressions for the co-ordinates (h, k) of the centre of curvature.

Solution.

Recall,

i) $\Rightarrow R = \dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}$

$x = \cos t + t \sin t$

$y = \sin t - t \cos t$

$\dfrac{dx}{dt} = -\cancel{\sin t} + t\cos t + \cancel{\sin t}(1)$

$\dfrac{dx}{dt} = t \cos t$

$\dfrac{dy}{dt} = \cancel{\cos t} + t \sin t - \cancel{\cos t}(1)$

$\dfrac{dy}{dt} = t \sin t$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx} = \dfrac{\cancel{t}\sin t}{\cancel{t}\cos t} = \dfrac{\sin t}{\cos t}$ //

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{\sin t}{\cos t}\right) \cdot \frac{dt}{dx}$$

$$\Rightarrow \frac{V\, du/dx - U\, \frac{dv}{dx}}{V^2} \equiv \frac{V\, du/dt - U\, \frac{dv}{dt}}{V^2}$$

Let $U = \sin t$

$V = \cos t$

$du/dt = \cos t$

$dv/dt = -\sin t$

$$\frac{d^2y}{dx^2} = \frac{\cos^2 t - (-\sin^2 t)}{\cos^2 t} \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$$

Recall from trigonometric Identities that : $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{t\cos^3 t}$$

Since
$$k = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$k = \left[1 + \left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2} \div \frac{d^2y}{dx^2}$$

$$k = \left[\frac{1}{1} + \frac{\sin^2 t}{\cos^2 t}\right]^{3/2} \times \frac{t\cos^3 t}{1}$$

$$R = \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right]^{3/2} \times \frac{t\cos^3 t}{1}$$

$$R = \frac{1}{(\cos^2 t)^{3/2}} \times \cancel{t\cos 3}\ t\cos^3 t$$

$$R = \frac{t\,\cancel{\cos^3 t}}{\cancel{\cos^3 t}}$$

$$R = t$$

∴ The expression for the radius of curvature (R) is t.

ii) (h, k)

Recall; $h = x_1 - R\sin\theta$ ... ①

$k = y_1 + R\cos\theta$ ... ②

$R = t$; $\theta = t$

$x_1 = \cos t + t\sin t$

$y_1 = \sin t - t\cos t$

Substituting for θ, x_1, y_1 & R in equation ① and ②

$h = \cos t + \cancel{t\sin t} - \cancel{t\sin t}$

$h = \cos t$

$k = \sin t - \cancel{t\cos t} + \cancel{t\cos t}$

$k = \sin t$

∴ The expression for the co-ordinates (h, k) of the centre of curvature is $(\cos t, \sin t)$.