1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

// convert equation into an homogeneous equation

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m^2 + m - 2m - 2 = 0$

$m(m+1) - 2(m+1) = 0$

$(m+1)(m-2) = 0$

$m_1 = -1$ and $m_2 = 2$

$y = Ae^{-x} + Be^{2x}$ // C.F.

$y = C$

$\frac{dy}{dx} = 0 \quad ; \quad \frac{d^2y}{dx^2} = 0$

$0 - 0 - 2C = 8$

$-2C = 8$

$C = 8/-2 = -4$

$y = -4$ // P.I

$y = Ae^{-x} + Be^{2x} - 4$ // G.S

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

// convert equation into an homogeneous equation

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \sqrt{4} = \pm 2$

$y = A\cos 2x + B\sin h 2x$ // C.F.

$y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$; $\frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

// divide equation by e^{3x}

$5C = 10$

$C = \frac{10}{5} = 2$

$y = 2e^{3x}$ // P.I.

$y = A\cos h 2x + B\sin h 2x + 2e^{3x}$ // G.S.

3. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

// convert equation into an homogeneous equation

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)$ twice $\therefore m_1 = m_2 = -1$

$y = e^{-x}(A + Bx)$ // C.F.

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$; $\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

// divide equation by e^{-2x}

$C = 1$

$y = e^{-2x}$ // P.I.

$y = e^{-x}(A + Bx) + e^{-2x}$ // G.S

4. $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

// convert equation into an homogeneous equation

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \sqrt{-25} = \sqrt{-1}\sqrt{25} = \pm 5j$

$y = A\cos 5x + B\sin 5x$ // C.F.

$y = Ax^2 + Bx + C$

$\frac{dy}{dx} = 2Ax + B \qquad \frac{d^2y}{dx^2} = 2A$

$2A + 25(Ax^2 + Bx + C) = 5x^2 + x$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$25A = 5$

$A = 5/25 = 1/5$

$25B = 1$

$B = 1/25$

$2A + 25C = 0$

$2(1/5) + 25C = 0$

$2/5 + 25C = 0$

$25C = -2/5$

$C = \frac{-2}{5} \times \frac{1}{25} = \frac{-2}{125}$

$y = 1/5x^2 + 1/25x - 2/125$ // P.I.

$y = A\cos 5x + B\sin 5x + 1/5x^2 + 1/25x - 2/125$ // G.S.

5. $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = 4\sin x$

// convert equation into an homogenous equation

$\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)$ twice $\quad \therefore m_1 = m_2 = 1$

$y = e^x(A + Bx)$ // C.F.

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x \quad , \quad \frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$-C\cos x - 2D\cos x + C\cos x - D\sin x + 2C\sin x + D\sin x = 4\sin x$

$\cos x(-C - 2D + C) + \sin x(-D + 2C + D) = 4\sin x$

$-C - 2D + C = 0 \Rightarrow 2D = 0 \Rightarrow D = 0$

$-D + 2C + D = 4 \Rightarrow 2C = 4 \Rightarrow C = 4/2 = 2$

$y = 2\cos x + 0\sin x = 2\cos x + 0 = 2\cos x$ // P.I.

$y = e^x(A + Bx) + 2\cos x$

6. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$

//convert equation into an homogeneous equation

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$

$a = 1, \; b = 4, \; c = 5$

$m = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1} \Rightarrow m = \frac{-4 \pm \sqrt{-4}}{2} \Rightarrow m = \frac{-4 \pm 2j}{2}$

$m = -2 \pm j$

$m_1 = -2 + j \qquad m_2 = -2 - j$

$y = e^{-2x}(C\cos x + D\sin x)$ //C.F.

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$; $\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4[-2Ce^{-2x}] + 5[Ce^{-2x}] = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 5C = 2$

$C = 2$

$y = 2e^{-2x}$ // P.I.

$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$ //G.S.

at $x = 0$ and $y = 1$

$1 = e^{-2(0)}[C\cos(0) + D\sin(0)] + 2e^{-2(0)}$

$1 = 1(C + 0) + 2$

$1 = C + 2$

$C = 1 - 2 = -1$

$\frac{dy}{dx} = [e^{-2x}(-C\sin x + D\cos x)] + [-2e^{-2x}(C\cos x + D\sin x)] - 4e^{-2x}$

when $\frac{dy}{dx} = -2$; $x = 0$

$-2 = (D) + (-2C) - 4$

$-2 = D - 2C - 4$

$D - 2C = -2 + 4$

$D = -2 + 4 + 2(-1)$

$D = -2 + 4 - 2 = 0$

$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$

$y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-2x}$

$y = e^{-2x}(-\cos x + 0) + e^{-2x}(2)$

$y = e^{-2x}(2 - \cos x)$ // P.S.

1. $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

Convert equation into an homogeneous equation

$3m^2 - 2m - 1 = 0$

$m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times -1}}{2 \times 3} \Rightarrow m = \frac{2 \pm \sqrt{4+12}}{6} \Rightarrow m = \frac{2 \pm 4}{6}$

$m = \frac{2+4}{6} \Rightarrow m = \frac{1+2}{3}$

$m_1 = \frac{1+2}{3} = \frac{3}{3} = 1$ or $m_2 = \frac{1-2}{3} = \frac{-1}{3}$

$m_1 = 1$ or $m_2 = -1/3$

$y = Ae^x + Be^{-1/3x}$ // C.F.

$y = Cx + D$

$\frac{dy}{dx} = C \quad ; \quad \frac{d^2y}{dx^2} = 0$

$3(0) - 2C - (Cx + D) = 2x - 3$

$-2C - Cx - D = 2x - 3$

$-C = 2$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$D = 4 + 3 = 7$

$y = -2x + 7$ // P.I

$y = Ae^x + Be^{-1/3x} - 2x + 7$

9. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

// convert equation into a homogenious equation

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 2m - 4m + 8 = 0$

$m(m-2) - 4(m-2) = 0$

$(m-2)(m-4) = 0$

$m_1 = 2$ and $m_2 = 4$

$y = Ae^{2x} + Be^{4x}$ // C.F.

$y = Cxe^{4x}$

$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$16Cxe^{4x} - 24Cxe^{4x} + 8Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$2C = 8$

$C = 8/2 = 4$

$y = 4xe^{4x}$ // P.I.

$y = Ae^{2x} + Be^{4x} + 4xe^{4x}$