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15/ENG01/005
CHEMICAL ENGINEERING.

1 $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Solution

$m^2 - m - 2 = 8$

$(m^2 - 2m) + 1(m - 2)$

$m(m-2) + 1(m-2)$

$(m+1)(m-2)$

$m_1 = -1$ and $m_2 = 2$

$\therefore C.F : y = Ae^{-x} + Be^{2x}$

P.I : $y = C$

$dy/dx = 0$

$d^2y/dx^2 = 0$

$\therefore 0 - 0 - 2(c) = 8$

$-2c = 8$

$c = -4$.

G.S : $y = Ae^{-x} + Be^{2x} - 4$

2 $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Solution.

$m^2 - 4 = 0$

$m = \pm 4 \quad m^2 = +4$

$m = \pm\sqrt{4}$

$m = \pm 2$

C.F : $y = A\cosh 2x + B\sinh 2x$

P.I $y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$d^2y/dx^2 = 9Ce^{3x}$

~~$9Ce^{3x} - 4[Ce^{3x}] = 10e^{3x}$~~

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$5C = 10$

$C = 10/5$

$C = 2$

G.S: $y = A\cosh 2x + B\sinh 2x + 2e^{3x}$

3 $d^2y/dx^2 + 2\,dy/dx + y = e^{-2x}$

Solution

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m[m^2 + m] + 1[m+1]$

$(m+1)(m+1)$.

$\therefore m_1 = -1$ and $m_2 = -1$

C.F: $y = e^{-x}[A + Bx]$

P.I: $y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$

$d^2y/dx^2 = 4Ce^{-2x}$

$4Ce^{-2x} + 2[-2(e^{-2x}] + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} + -4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = \dfrac{e^{-2x}}{e^{-2x}}$

$C = 1$ $\therefore$ P.I $= y = e^{-2x}$

G.S: $y = e^{-x}[A+Bx] + e^{-2x}$ i.e

4 $d^2y/dx^2 + 25y = 5x^2 + x$

Solution

$m^2 + 25 = 0$

$m = \pm\sqrt{-25}$

5 $\frac{d^2y}{dx^2} - 2\,dy/dx + y = 4\sin x$

Solution

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m[m-1] - 1[m-1] = 0$

$m - 1 = 0$

$m = 1$ twice

$\therefore m_1 = m_2 = 1$

C.F: $y = e^x[A + Bx]$

P.I: $y = C\cos x + D\sin x$

$dy/dx = -C\sin x + D\cos x$

$d^2y/dx^2 = -C\cos x - D\sin x$

$-C\cos x - D\sin x - 2[-C\sin x + D\cos x] + C\cos x + D\sin x = 4\sin x$

$\cos x[-C - 2D + C] + \sin x[-D + 2C + D] = 4\sin x$

$-2D = 0 \quad \ldots (1)$

$2C = 4 \quad \ldots (2)$

From eqn (1),

$D = 0$

and from eqn (2)

$C = 4/2$

$C = 2$

P.I $= y = 2\cos x$

G.S: $y = e^x[A + Bx] + 2\cos x$

4 $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

solution

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{-25}$

$m = \pm 5i$

C.F : $y = C\cos 5x + D\sin 5x$

P.I : $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25[Cx^2 + Dx + E] = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$2C + 25E + 25Dx + 25Cx^2 = 5x^2 + x$

$2C + 25E = 0$ --- ①

$25Dx = 1$ --- ②

$25Cx^2 = 5x^2$ --- ③

From eqn ③

$C = \frac{5x^2}{25x^2}$

$C = 1/5$

From ②

$D = 1/25$

$2C + 25E = 0$

$2[1/5] + 25E = 0$

$2/5 + 25E = 0$

$25E = -2/5$

$E = -2/125$

$y = Cx^2 + Dx + E$

P.I : $y = 1/5x^2 + 1/25x - 2/125$

$G.S = C\cos 5x + D\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$= C\cos 5x + D\sin 5x + \frac{1}{25}[25x^2 + 5x - 2]$

6 $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$

$a = 1, \quad b = 4, \quad c = 5$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm 2i}{2}$

$m = -2 \pm i$

$\beta = 1, \quad \alpha = -2$

$y = e^{-2x}[A\cos x + B\sin x]$.

P.I : $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5[Ce^{-2x}] = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 5C = 2$

$C = 2$

P.I : $y = 2e^{-2x}$

G.S : $y = e^{-2x}[A\cos x + B\sin x] + 2e^{-2x}$

$\frac{dy}{dx} = -2e^{-2x}[A\cos x + B\sin x] + e^{-2x}[-A\sin x + B\cos x] + 4$

ie using $A\cos x + B\sin x$ as v and $u = e^{-2x}$

Since $\frac{dy}{dx} = 2$, $x = 0$, $y = 1$

$1 = e^{-2(0)}[A\cos 0 + B\sin 0] + 2e^{-2(0)}$

$1 = A + 2$

$A = -1$

$-2 = -2e^{-2(0)}[A\cos 0 + B\sin 0] + e^{-2(0)}[-A\sin 0 + B\cos 0] - 4e^{-2(0)}$

$-2 = -2[A + 0] + 1[0 + B] - 4$

$-2 = -2A + B - 4$

$-2A + B = 2$

$-2(-1) + B = 2$

$2 + B = 2$

$B = 0$

P.S. $y = e^{-2x}[-\cos x + 0\sin x) + 2e^{-2x}$

$y = e^{-2x} - \cos x + 2e^{-2x}$

$y = e^{-2x}[2 - \cos x]$

7 $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

~~$3[m^2 - m]$~~

$3m[m - 1] + 1[m - 1] = 0$

$3m + 1 = 0 \qquad m - 1 = 0$

$m = -1/3 \quad$ or $\quad m = 1$

C.F. $y = Ae^{x} + Be^{-x/3}$

P.I. : $y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$3(0) - 2(C) - 1[Cx + D] = 2x - 3$

$-2C - D : -Cx = 2x - 3$

$-Cx = 2x$

$-C = 2$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$3 + 4 = D$

$7 = D$

P I : $y = -2x + 7$

G.S : $y = Ae^{x} + Be^{-x/3} - 2x + 7$

Q6 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m[m-4] - 2[m-4] = 0$

$(m-4)(m-2) = 0$

$m - 4 = 0 \qquad m - 2 = 0$

$m = 4 \qquad m = 2$

C.F : $y = Ae^{4x} + Be^{2x}$

P.I : $y = Cxe^{4x}$

$y = uv \; ; \; u = Cx \quad v = e^{4x}$

$\frac{dy}{dx} = v\,du + u\,dv$

$du = C$

$dv = 4e^{4x}$

$\frac{dy}{dx} = e^{4x} \cdot C + Cx \cdot 4e^{4x} = Ce^{4x} + 4Cxe^{4x}$

$\frac{d^2y}{dx^2} = 4Ce^{4x} + 4Ce^{4x} + 16Cxe^{4x} = 8Ce^{4x} + 16Ce^{4x}$

$8Ce^{4x} + 16Cxe^{4x} - 6[Ce^{4x} + 4Cxe^{4x}] + 8[Cxe^{4x}] = 8e^{4x}$

$8C + 16Cx - 6C - 24Cx + 8Cx = 8$

$8C - 6C + 16Cx - 24Cx + 8Cx = 8$

$8C - 6C = 8$

$2C = 8$

$C = 4$

P.I : $y = Cxe^{x}$

$y = 4xe^{x}$

G.S : $y = Ae^{4x} + Be^{2x} + 4xe^{x}$.