Ayeni Ayobami Opeyemi
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ENG 382 Assignment 2.

10. $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

Soln

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4^2 - 4\times1\times5}}{2\times1} = \frac{-4 \pm \sqrt{16-20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm \sqrt{-1}\sqrt{4}}{2} = \frac{-4 \pm 2j}{2}$

$m = -2 \pm j$

$y = e^{-2\theta}[C\cos\theta + D\sin\theta]$ — CF

Particular integral

$y = A\cos\theta + B\sin\theta$

$\frac{dy}{d\theta} = -A\sin\theta + B\cos\theta$

$\frac{d^2y}{d\theta^2} = -A\cos\theta - B\sin\theta$

$\therefore [-A\cos\theta - B\sin\theta] + 4[-A\sin\theta + B\cos\theta] + 5[A\cos\theta + B\sin\theta] = 6\sin\theta$

$-A\cos\theta - B\sin\theta - 4A\sin\theta + 4B\cos\theta + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$

$-A\cos\theta + 4B\cos\theta + 5A\cos\theta - B\sin\theta - 4A\sin\theta + 5B\sin\theta = 6\sin\theta$

Comparing Coefficients

$-A + 4B + 5A = 0$

$4A + 4B = 0$ — eq(1)

$-B - 4A + 5B = 6$

$-4A + 4B = 6$ — eq(2)

adding eq(1) & eq(2)

$8B = 6$

$B = \frac{3}{4}$

$4A + 4(\frac{3}{4}) = 0$

$4A = -3 \quad \therefore A = -3/4$

$$y = \frac{-3}{4}\cos\theta + \frac{3}{4}\sin\theta \quad - (P.I)$$

G.S:

$$y = e^{-2\theta}(C\cos\theta + D\sin\theta) - \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$$

(ii) at $\theta = \infty$

and $\frac{dy}{d\theta} = 0$

$$\frac{dy}{d\theta} = e^{-2\theta}[-C\sin\theta + D\cos\theta] + [C\cos\theta + D\sin\theta](-2e^{-2\theta}) + \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$$

at $\theta = \infty$ and $\frac{dy}{d\theta} = 0$

$$0 = e^{-2\times\infty}[-C\sin\infty + \quad$$

$$0 = \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$$

$$-\frac{3}{4}\cos\theta = \frac{3}{4}\sin\theta$$

$$-\cos\theta = \sin\theta$$

divide both sides by $-\cos\theta$

$$\frac{\sin\theta}{-\cos\theta} = 1$$

$$-\tan\theta = 1$$

$$\tan\theta = -1$$

$$\theta = \tan^{-1}(-1)$$

$$= -45^\circ$$

2. $$EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$$

Sol

$$EI\,m^2 = 0$$

$$m^2 = 0$$

$$m = \pm\sqrt{0}$$

$$m = \pm 0$$

$y = e^{0x}(A + Bx)$

$y = A + Bx$ --- CF.

To obtain particular integral

$y = Px^2 + Qx^3 + Rx^4$

$\frac{dy}{dx} = 2Px + 3Qx^2 + 4Rx^3$

$\frac{d^2y}{dx^2} = 2P + 6Qx + 12Rx^2$

$EI\left[2P + 6Qx + 12Rx^2\right] = \frac{W}{2}(L - x)^2$

$2PEI + 6QEIx + 12REIx^2 = \frac{W}{2}(L^2 - 2Lx + x^2)$

$4PEI + 12QEIx + 24REIx^2 = W(L^2 - 2Lx + x^2)$

$24REI = W$

$R = \frac{W}{24EI}$ --- (1)

$12QEI = -2WL$

$Q = \frac{-2WL}{12EI} = \frac{-WL}{6EI}$ --- (2)

$4PEI = WL^2$

$P = \frac{WL^2}{4EI}$

$y = \left[\frac{WL^2}{4EI}\right]x^2 - \left[\frac{WL}{6EI}\right]x^3 + \left[\frac{W}{24EI}\right]x^4$

$= \frac{WL^2x^2}{4EI} - \frac{WLx^3}{6EI} + \frac{Wx^4}{24EI}$

$$= \frac{6WL^2x^2 - 4WLx^3 + Wx^4}{24EI}$$

$$= \frac{W}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right] \quad -- \quad PI$$

G.S

$$y = A + Bx + \frac{W}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

at $y = 0,\ x = 0,\ \frac{dy}{dx} = 0$

$$0 = A + B(0) + \frac{W}{24EI}\left[6L^2(0) - 4L(0) + 0\right]$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{W}{24EI}\left[12L^2x - 12Lx^2 + 4x^3\right]$$

Then

$$0 = B + \frac{W}{24EI}\left[12(0) - 12(0) + 4(0)\right]$$

$$B = 0$$

Particular Solution

$$y = \frac{W}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = \frac{Wx^2}{24EI}\left[6L^2 - 4Lx + x^2\right]$$

$$y = \frac{Wx^2}{24EI}\left[x^2 - 4Lx + 6L^2\right]$$

When $x = L$

$$y = \frac{WL^2}{24EI}\left[L^2 - 4L^2 + 6L^2\right] = \frac{WL^2}{24EI}\left[3L^2\right]$$

$$y = \frac{WL^4}{8EI}$$