Assignment 1

(1) $m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2)$

$(m+1)(m-2)$

$m_1 = -1$

$m_2 = 2$

$y = Ae^{-x} + Be^{2x}$ – Complementary function

Particular Integral ⇒ $y = C$

$dy/dx = 0$

$\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2C = 8$

$-2C = 8$

$C = -4$

P.I $y = -4$

General Solution

$y = Ae^{-x} + Be^{2x} - 4$

(2) $d^2y/dx^2 - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$m^2 - 2m + 2m - 4 = 0$

$m(m-2) + 2(m-2) = 0$

$(m+2)(m-2) = 0$

$m_1 = -2$

$m_2 = +2$

$y = Ae^{-2x} + Be^{2x}$

P.I ⇒ $y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$d^2y/dx^2 = 9Ce^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$e^{3x}(9C - 4C) = 10e^{3x}$

$e^{3x}(5C) = 10e^{3x}$

$5C = 10$

$C = 2$

P.I ⇒ $y = 2e^{3x}$

∴ General Solution

$y = Ae^{-2x} + Be^{2x} + 2e^{3x}$

Q. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1)$

$m_1 = m_2 = -1$

$y = e^{-x}(A + Bx)$

P.I $\Rightarrow y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$

$d^2y/dx^2 = +4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$C = 1$

P.I $\Rightarrow y = e^{-2x}$

General Solution

$y = e^{-x}(A + Bx) + e^{-2x}$

Q. $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$m^2 + 25 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0 \pm \sqrt{-4(1)(25)}}{2}$

$= \frac{\pm\sqrt{-100}}{2}$

$= \pm j\sqrt{100}$

$= \pm j10$

x_1

$= \pm 5j$

C.F

$y = C\cos 5x - D\sin 5x$

P.I $\Rightarrow y = Cx^2 + Dx + E$

$dy/dx = 2(x+1)$

$d^2y/dx^2 = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 + 25Dx + 25E + 2C = 5x^2 + x$

$25C = 5$ $\quad 25E + 2C = 0$

$C = 1/5$ $\quad 25E + \frac{2}{5} = 0$

$25D = 1$ $\quad 25E = -2/5$

$D = 1/25$ $\quad E = -2/5 \div 25$

$E = \frac{-2}{125}$

P.I

$y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

General Solution

$y = C\cos 5x - D\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

P.I.

$y = 2\cos x$

$\therefore$ G.S

$y = e^x(A+Bx) + 2\cos x$

(5) $d^2y/dx^2 - 2\,dy/dx + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$m^2 - 1m - 1m + 1 = 0$

$m(m-1) - 1(m-1)$

$(m-1)(m-1) = 0$

$m = 1$

C.F

$y = e^x(A+Bx)$

P.I. $y = C\cos x + D\sin x$

$dy/dx = -C\sin x + D\cos x$

$d^2y/dx^2 = -C\cos x - D\sin x$

$(-C\cos x - D\sin x) - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$(-C - 2D + C)\cos x + (-D + 2C + D)\sin x = 4\sin x$

$-2D\cos x + 2C\sin x = 4\sin x$

$2C = 4$

$C = 2$

$-2D = 0$

$D = 0$

⑥ $d^2y/dx^2 + 4dy/dx + 5y = 2e^{-2x}$

$m^2 + 4m + 5 = 0$

$m(m+4) - 5(m+0) = 0$

$\frac{-4 \pm \sqrt{4^2 - 4(1)(0)}}{2}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm 2j}{2}$

$= -2 \pm j$

$= e^{-2x}(C\cos x + D\sin x)$

P.I

$y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$

$d^2y/dx^2 = 4Ce^{-2x}$

$4Ce^{-2x} + 8Ce^{-2x} + 5Ce^{-4} = 2e^{-2x}$

$Ce^{-4} = 2e^{-4}$

$C = 2$

$y = 2e^{-2x}$

GS

$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$

If $y = 1$, $x = 0$

$1 = e^{-2(0)}(C\cos(0) + D\sin(0)) + 2e^{-2(0)}$

$1 = C + 2$

$C = 1 - 2$

$C = -1$

$\frac{dy}{dx} = (e^{-2x})(-\sin x + D\cos x) + (-2e^{-2x})(-\cos x + D\sin x)$

$-2e^{-2x}$

$-2 = (0)(\cos(0)) + (-2)(C\cos(0))$

$-2e^{-2(0)}$

$-2 = D - 2C - 2$

$-2 + 2 = D - 2C$

$0 = D - 2C$

$D = 2C$

$D = 2(-1)$

$D = -2$

$y = e^{-2x}(-\cos x - 2\sin x) + 2e^{-2x}$

(2) $3\frac{d^2y}{dx^2} - 2\,dy/dx - y = 2x - 3$

$3m^2 - 2m - 1$

~~$3m^2 - 2m$~~

$3m^2 - 3m + m - 1$

$3m(m-1) + 1(m-1)$

$(3m+1)(m-1)$

$m = -1/3, \quad m = 1$

C.F

$y = Ae^{-\frac{1}{3}x} + Be^{x}$

P.I

$y = Cx + D$

$dy/dx = C$

$d^2y/dx^2 = 0$

$3(0) - 2C - y(Cx + D) = 2x - 3$

$-2C - Cx - D = 2x - 3$

$-C = 2$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$-D = -3 - 4$

$D = 7$

P.I

$y = -2x + 7$

G.S

$y = Ae^{-\frac{1}{3}x} + Be^{x} - 2x + 7$

(6) $d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4) = 0$

$(m-2)(m-4) = 0$

$m = 2, \quad m = 4$

$y = Ae^{2x} + Be^{4x}$

P.I $\Rightarrow$ $y = Ce^{4x}$

$dy/dx = 4Ce^{4x}$

$d^2y/dx^2 = 16Ce^{4x}$

Won't work.

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

P.T.O

(5) Cont'd

$y = Cxe^{4x}$

$\frac{dy}{dx} = $

$\frac{dy}{dx} = C\left(4e^{4x}\right)xCe^{4x}$

$= 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 4Cx\left(4e^{4x}\right) + 4C\left(e^{4x}\right)$

$+ 4Ce^{4x}$

$= 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$= 16Cxe^{4x} + 8Ce^{4x}$

$\therefore 16Cxe^{4x} + 8Ce^{4x}$

$-6\left(4Cxe^{4x} + Ce^{4x}\right)$

$+ 8Cxe^{4x} = 8e^{4x}$

$16Cxe^{4x} + 8Ce^{4x} - 24Cxe^{4x}$

$-6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$16Cx + 8C - 24Cx - 6C + 8Cx = 8$

$16Cx + 8Cx - 24Cx + 8C - 6C = 8$

$24Cx - 24Cx + 8C - 6C = 8$

$2C = 8$

$C = 4$

P.I

$y = 4xe^{4x}$

G.S

$y = Ae^{2x} + Be^{4x} + 4xe^{4x}$