15/ENG04/012

A3EE3 Fiyin Foluwa Andrew

ENG281 Assignment

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Soln

$m^2 + m - 2 = 0$

$\frac{+1 \pm \sqrt{-1^2 - 4 \times 2}}{2}$

$m = \frac{+1 \pm \sqrt{9}}{2}$

$m = \frac{+1+3}{2}, \frac{+1-3}{2}$

$m = 2$ or $m = -1$

$y' = Ae^{-2x} + Be^{-x}$ — C·F

$y = C$

$\frac{dy}{dx} = 0 \quad \frac{d^2y}{dx} = 0$

$0 - 0 - 2c = 8$

$-2c = 8$

$\frac{8}{-2} = c$

$c = -4 \qquad y = -4$

[redacted] gen soln: $y = A\cos 3x +$

General Solution

$y = Ae^{2x} + Be^{-x} - 4$

2. $\frac{d^2y}{dx} - 4y = 10e^{3x}$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = 4 \quad m = \pm\sqrt{4}$

$m = \pm 2$

$y = e^0(C\cosh 2x + D\sinh 2x)$ Cf

For PI

$y = Ae^{3x}$

$\frac{dy}{dx} = 3e^{3x} \quad \frac{d^2y}{dx} = 9Ae^{3x}$

$9Ae^{3x} - 4(Ae^{3x}) = 10e^{3x}$

$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$A\cancel{e^{3x}} = \frac{10e^{3x}}{5e^{3x}} = 2$

$y = 2e^{3x}$

General Soln. $y = C\cosh 2x + D\sinh 2x + 2e^{3x}$

3

$\frac{d^2y}{dx^2} + 2\frac{dy}{} + y \; e^{-2x}$

Soln

$\frac{d^2y}{dx} - 2\frac{dy}{dx} + y = 0$

$m^2 + 2m + 1 = 0$

$\frac{-2 \pm \sqrt{2^2 - 4}}{2} = \frac{-2 \pm 0}{2} = -1$

$m_1 = m_2 = -1$

$y = e^{-x}(A + Bx)$ — C.F

For P.I

$\frac{dy}{dx} = -2Ce^{2x} \quad \frac{d^2y}{dx} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$Ce^{-2x}[4 - 4 + 1] = e^{-2x}$

$C = \frac{e^{-2x}}{e^{-2x}} = 1$

$\therefore y = e^{-2x}$ - - - PI

General soln

$y = e^{-x}(A + Bx) + e^{-2x}$

4

$\frac{d^2y}{dx} + 25y = 5x + x$

$m^2 + 25 = 0$

$m^2 = -25 \therefore m \pm \sqrt{25} \quad m = \pm j5$

$y \; C\cos 5x + D\sin 5x$ — Cf

For PI

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D \quad \frac{d^2y}{dx} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 + 25Dx + 25E + 2C = 5x^2 + x$

$25C = 5$

$C = \frac{1}{5}$ — (1)

$25D = 1$

$D = 1/25$ — (1)

$25E + 2C = 0$

$\frac{2}{5} \cdot (25) = E$

$E = 2/125$

$y = \frac{1}{5}x + 1/25 x + 2/125$

gen Soln $= y\{C\cos 5x + D\sin 5x\} + \frac{1}{5}x^2 + \frac{1}{25}x + \frac{2}{125}$

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5. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$\frac{-2 \pm \sqrt{4-4}}{2} = \frac{-2 \pm 2}{2} = 1$

$m_1 = m_2 = 1$

$y = e^{x}(A + Bx)$ — CF

For PI $y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -\cos x$

$-\cos x - 2(-\sin x + D\cos x) + C\cos x$

$+ D\sin x = 4\sin x$

$-\cos x \;\; 2\sin x + 2D\cos x + C\cos x$

$+ D\sin x = 4\sin x$

$-D + 2C + D = 4$

$2C = 4 \quad C = 2$

$-C - 2D + C = 0$

$2D = 0 \quad D = 0$

$y = 2\cos x + 0\sin x = 2\cos x$

$GS = y = e^{x}(A + Bx) + 2\cos x$

6. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

Soln

$\frac{-b \pm \sqrt{4^2 - 4 \cdot 1 \cdot 5}}{2}$

$m = \frac{-4 \pm \sqrt{-1}\sqrt{4}}{2} = \frac{-4 \pm j2}{2}$

$m_1 = -2 + j \quad m_2 = -2 - j$

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x} \quad \frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$C = \frac{2e^{-2x}}{e^{-2x}} = 2$

$y = 2e^{-2x}$ — P.I

G.S $y = e^{-2x}(C\cos 2x + D\sin 2x)$

$+ 2e^{-2x}$

@ $x = 0$ $y = 1$ $dy/dx = -2$

$1 = e^{0}(C\cos 0 \;\; D\sin 0 + 2e^{0}$

$1 = C + 2$

$C = -1$

$$\frac{dy}{dx} = e^{-2x}(-C\sin x + D\cos x) + [C\cos x + D\sin x]$$

$$\times -2e^{-2x} - 4e^{-2x}$$

at $x = 0$ $y = 1$ $\frac{dy}{dx} = -2$

$$-2 = e^{0}[-C\cancel{\sin 0}^{0} + D\cos 0] + [C\cos 0 + \cancel{D\sin 0}^{0}] \cdot -2e^{0} - 4e^{0}$$

$$-2 = D - 2C - 4$$

Since $C = -1$

$$-2 = D - 2(-1) - 4$$

$$-2 = D + 2 - 4$$

$$-2 + 2 = D$$

$$D = 0$$

P.S = $y = e^{-2x}[-\cos x] + 2e^{-2x}$

$$\therefore y = -e^{-2x}\cos x + 2e^{-2x}$$

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$$\frac{dy}{dx} = C$$

$$\frac{d^2y}{dx^2} = 0$$

$$0 - 2C - (Cx + D) = 2x - 3$$

$$-2C - Cx - D = 2x - 3$$

Comparing coefficients

$$-C = 2$$

$$C = -2$$

$$-2C - D = -3$$

$$-2(-2) - D = -3$$

$$4 - D = -3$$

$$D = 7$$

$$y = -2x + 7$$

$$G.S : y = Ae^{-\frac{1}{3}x} + Be^{x} - 2x + 7$$

8. $$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$$

$$m^2 - 6m + 8 = 0$$

$$m^2 - 2m - 4m + 8 = 0$$

7. $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$

$3m^2 - 2m - 1 = 0$

$\cancel{3m^2 + \frac{1}{3}m - m - 1 = 0}$

By F

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(3m+1)(m-1) = 0$

$m_1 = -\frac{1}{3} \quad m_2 = 1$

$y = Ae^{-\frac{1}{3}x} + Be^{x}$ — C.F

to obtain P.I

$y = Cx + D$

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$m(m-2) - 4(m-2) = 0$

$(m-4)(m-2) = 0$

$m_1 = 4 \quad m_2 = 2$

$y = Ae^{4x} + Be^{2x}$ — — C.F

1- obtain P.S

$y = Cxe^{4x}$

$\frac{dy}{dx} = C[x \cdot 4e^{4x} + e^{4x}]$

$= 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 4C[x4e^{4x} + e^{4x}] + 4Ce^{4x}$

$= 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$16Cxe^{4x} + 8Ce^{4x} - 6[4Cxe^{4x} + Ce^{4x}] +$
$8[Cxe^{4x}] = 8e^{4x}$

$16Cxe^{4x} + 8Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x}$
$= 8e^{4x}$

$Cxe^{4x}(\overbrace{16 - 24 + 8}^{0}) + Ce^{4x}(8-6) = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$C = \frac{8e^{4x}}{2e^{4x}} = 4$

$y = 4xe^{4x}$

G.S: $y = Ae^{4x} + Be^{2x} + 4xe^{4x}$