NDUBUISI DAVID KELECHI
15/ENG01/008
CHEMICAL ENGINEERING
ENG 381
ASSIGNMENT

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

First we get the complementary function

C.F: $d^2y/dx^2 - dy/dx - 2y = 0$

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2) = 0$

$m_1 = -1 \qquad m_2 = 2$

C.F: $y = Ae^{-x} + Be^{2x}$

Then we get the particular integral

P.I: $f(x) = 8$

$y = c$

$dy/dx = 0$

$d^2y/dx^2 = 0$

$(0) - (0) - 2(c) = 8$

$-2c = 8$

$c = 8/-2$

$c = -4$

P.I: $y = -4$

General solution = Complementary function + Particular integral.

G.S: $y = Ae^{-x} + Be^{2x} - 4$

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

C.F: $d^2y/dx^2 - 4y = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2$

C.F: $y = C\cosh 2x + D\sinh 2x$

P.I: $f(x) = 10e^{3x}$

$y = Ce^{3x}$

$dy/dx = 3Ce^{3x}$

$d^2y/dx^2 = 9Ce^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$C = \frac{10}{5}$

$C = 2$

P.I: $y = 2e^{3x}$

G.S: $y = C\cosh 2x + D\sinh 2x + 2e^{3x}$

3. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

C.F: $d^2y/dx^2 + 2\,dy/dx + y = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1$ (twice)

C.F: $y = e^{-x}(A + Bx$

P.I: $f(x) = e^{-2x}$

$y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$

$$\frac{d^2y}{dx^2} = 4ce^{-2x}$$

$$4ce^{-2x} + 2(-2ce^{-2x}) + ce^{-2x} = e^{-2x}$$

$$4ce^{-2x} - 4ce^{-2x} + ce^{-2x} = e^{-2x}$$

$$ce^{-2x} = e^{-2x}$$

$$c = 1$$

P.I : $y = e^{-2x}$

G.S : $y = e^{-x}(A+Bx) + e^{-2x}$

4. $$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

C.F : $\frac{d^2y}{dx^2} + 25y = 0$

$$m^2 + 25 = 0$$

$$m^2 = -25$$

$$m = \pm\sqrt{-25}$$

$$m = \pm\sqrt{-1} \times \sqrt{25}$$

$$m = \pm j5$$

C.F : $y = C\cos 5x + D\sin 5x$

P.I : $f(x) = 5x^2 + x$

$$y = Cx^2 + Dx + E$$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$$

$25C = 5 \quad \therefore \quad C = \frac{5}{25} = \frac{1}{5}$

$25D = 1 \quad \therefore \quad D = \frac{1}{25}$

$$2C + 25E = 0$$

$$2\left(\frac{1}{5}\right) + 25E = 0$$

$$\frac{2}{5} = -25E$$

$$E = \frac{2}{-125}$$

P.I : $y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

Scanned by CamScanner

G.S:
$$y = C\cos 5x + D\sin 5x + \frac{1}{125}(25x^2 + 5x - 2)$$

5. $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

C.F: $d^2y/dx^2 - 2\,dy/dx + y = 0$

$$m^2 - 2m + 1 = 0$$
$$m^2 - m - m + 1 = 0$$
$$m(m-1) - 1(m-1) = 0$$
$$(m-1)(m-1) = 0$$
$$m_1 = m_2 = 1$$

C.F: $y = e^x(A + Bx)$

P.I: $f(x) = 4\sin x$
$$y = A\cos x + B\sin x$$
$$dy/dx = -A\sin x + B\cos x$$
$$d^2y/dx^2 = -A\cos x - B\sin x$$

$$(-A\cos x - B\sin x) - 2(-A\sin x + B\cos x) + (A\cos x + B\sin x) = 4\sin x$$

$$-A\cos x - B\sin x + 2A\sin x - 2B\cos x + A\cos x + B\sin x = 4\sin x$$

$$(-A - 2B + A)\cos x + (-B + 2A + B)\sin x = 4\sin x$$

$$-2B\cos x + 2A\sin x = 4\sin x$$

$-2B = 0 \quad \therefore \quad B = -\frac{0}{2} = 0$

$2A = 4 \quad \therefore \quad A = 4/2 = 2$

$$y = 2\cos x + 0(\sin x)$$

P.I: $y = 2\cos x$

G.S: $y = e^x(A + Bx) + 2\cos x$

NDUBUISI DAVID KELECHI
15/ENG01/008
CHEMICAL ENGINEERING

6. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

C.F: $d^2y/dx^2 + 4\,dy/dx + 5y = 0$

$m^2 + 4m + 5 = 0$

$a = 1 \quad b = 4 \quad c = 5$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-4 \pm \sqrt{4^2 - 4(1\times5)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 4(5)}}{2}$

$\frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm \sqrt{-1} \times \sqrt{4}}{2} = \frac{-4 \pm j\sqrt{4}}{2}$

$= \frac{-4 \pm 2j}{2} = -2 \pm j$

C.F: $y = e^{-2x}(A\cos x + B\sin x)$

P.I: $f(x) = 2e^{-2x}$

Since e^{-2x} is already included in the C.F, we multiply by x

$y = Cxe^{-2x}$

$dy/dx = $ let $u = Cx \qquad v = e^{-2x}$

$du = C \qquad dv = -2e^{-2x}$

$udv + vdu$

$(Cx \cdot -2e^{-2x}) + (e^{-2x} \cdot C)$

$Ce^{-2x} - 2Cxe^{-2x}$

$\frac{d^2y}{dx^2} = -2Ce^{-2x} - \left[2Cx \cdot (-2e^{-2x}) + (e^{-2x}) \cdot 2C\right]$

where $u = 2Cx \qquad v = e^{-2x}$

$du = 2C \qquad dv = -2e^{-2x}$

$= -2Ce^{-2x} - [-4Cxe^{-2x} + 2Ce^{-2x}]$

$= -2Ce^{-2x} + 4Cxe^{-2x} - 2Ce^{-2x}$

$= 4Cxe^{-2x} - 4Ce^{-2x}$

$$(4Cxe^{-2x} - 4Ce^{-2x}) + 4(Ce^{-2x} - 2Cxe^{-2x}) + 5(Cxe^{-2x}) = 2e^{-2x}$$

$$4Cxe^{-2x} - 4Ce^{-2x} + 4Ce^{-2x} - 8Cxe^{-2x} + 5Cxe^{-2x} = 2e^{-2x}$$

$$Cxe^{-2x} = 2e^{-2x}$$

$$C = 2/x$$

$$\text{P.I: } y = \frac{2}{x} \cdot xe^{-2x}$$

$$y = 2e^{-2x}$$

$$\text{G.S: } y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$$

Given that at $x = 0$, $y = 1$ and $dy/dx = -2$

General solution: $y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$$1 = e^{-2(0)}[A\cos(0) + B\sin(0)] + 2e^{-2(0)}$$

$$1 = A + 2$$

$$A = 1 - 2$$

$$A = -1$$

$\frac{dy}{dx}$ = using product rule

$$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$$

$u = e^{-2x}$ $\quad$ $v = A\cos x + B\sin x$

$du = -2e^{-2x}$ $\quad$ $dv = -A\sin x + B\cos x$

$$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) + (A\cos x + B\sin x)(-2e^{-2x}) - 4e^{-2x}$$

$$-2 = e^{-2(0)}[-A\sin(0) + B\cos(0)] + [A\cos(0) + B\sin(0)](-2e^{-2(0)}) - 4e^{-2(0)}$$

$$-2 = B + A(-2) - 4$$

~~$-2 = B - 2A$~~

$-2 = B - 2A - 4$

since $A = -1$

$-2 = B - 2(-1) - 4$

$-2 = B + 2 - 4$

$B = 2 - 4 + 2$

$B = 4 - 4$

$B = 0$

$y = e^{-2x}(-\cos x + (0)\sin x) + 2e^{-2x}$

$\therefore y = e^{-2x}(-\cos x) + 2e^{-2x}$

7. $\frac{3d^2y}{dx^2} - \frac{2dy}{dx} - y = 2x - 3$

C.F: $3\,d^2y/dx^2 - 2\,dy/dx - y = 0$

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(m-1)(3m+1) = 0$

$m_1 = 1, \; m_2 = -1/3$

C.F: $y = Ae^{x} + Be^{-\frac{1}{3}x}$

P.I: $f(x) = 2x - 3$

$y = Cx + D$

$dy/dx = C$

$d^2y/dx^2 = 0$

$3(0) - 2(C) - (Cx + D) = 2x - 3$

$-2C - Cx - D = 2x - 3$

$-C = 2 \quad \therefore \quad C = -2$

$-2C - D = -3 \quad \therefore \quad -2(-2) - D = -3$

$4 - D = -3$

$4 + 3 = D$

$D = 7$

P.I: $y = -2x + 7$

G.S: $y = Ae^{x} + Be^{-\frac{1}{3}x} - 2x + 7$

NDUBUISI DAVID KELECHI
15/ENG01/008
CHEMICAL ENGINEERING

8. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8 = 8e^{4x}$

C.F: $d^2y/dx^2 - 6\,dy/dx + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4) = 0$

$(m-2)(m-4) = 0$

$m_1 = 2 \qquad m_2 = 4$

C.F: $y = Ae^{2x} + Be^{4x}$

P.I: $f(x) = 8e^{4x}$

Since e^{4x} is already included in the C.F, we multiply by x

$y = Cxe^{4x}$

dy/dx let $u = Cx \qquad v = e^{4x}$

$du = C \qquad dv = 4e^{4x}$

Using product rule $udv + vdu$

$(Cx \cdot 4e^{4x}) + (e^{4x} \cdot C)$

$4Cxe^{4x} + Ce^{4x}$

$= Ce^{4x} + 4Cxe^{4x}$

$\frac{d^2y}{dx^2}$. Using product rule to solve $4Cxe^{4x}$

$u = 4Cx \qquad v = e^{4x}$

$du = 4C \qquad dv = 4e^{4x}$

$4Ce^{4x} + [(4Cx \cdot 4e^{4x}) + (e^{4x} \cdot 4C)]$

$4Ce^{4x} + 16Cxe^{4x} + 4Ce^{4x}$

$= 16Cxe^{4x} + 8Ce^{4x}$

Putting it in the equation

$(16Cxe^{4x} + 8Ce^{4x}) - 6(Ce^{4x} + 4Cxe^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$= 16Cxe^{4x} + 8Ce^{4x} - 6Ce^{4x} - 24Cxe^{4x} + 8Cxe^{4x} = 8e^{4x}$

$$-8Cxe^{4x} + \cancel{8Ce} \; 8Cxe^{4x} + 8Ce^{4x} \cancel{+} - 6Ce^{4x} = 8e^{4x}$$

$$8Ce^{4x} - 6Ce^{4x} = 8e^{4x}$$

$$2Ce^{4x} = 8e^{4x}$$

$$C = \frac{8}{2}$$

$$C = 4$$

$$P.I : y = 4xe^{4x}$$

$$G.S : y = Ae^{2x} + Be^{4x} + 4xe^{4x}$$