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15/ENG06/056
Mechanical Engineering

ENG 381 Assignment 1

1) $d^2y/dx^2 - dy/dx - 2y = 8$

soln

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

$m^2 - m - 2 = 0$

$m^2 + m - 2m - 2 = 0$

$m(m+1) - 2(m+1) = 0$

$\therefore m = 2$ or $m = -1$

$\therefore y = Ae^{2x} + Be^{-x} \Rightarrow cf$

$y = c$

$dy/dx = 0$

$d^2y/dx^2 = 0$

$0 - 0 - 2c = 8$

$-2c = 8 \quad \therefore c = -4$

$\therefore y = -4$

General solution = C.F + PI

$\underline{y = Ae^{2x} + Be^{-x} - 4}$

2) $d^2y/dx^2 - 4y = 10e^{3x}$

$$\frac{d^2y}{dx^2} - 4y = 0$$

$m^2 - 4 = 0$

$m^2 = 4$

$\therefore m = \pm 2$

$y = C\cosh 2x + D\sinh 2x \Rightarrow cf$

$y = Ae^{3x}$

$dy/dx = 3Ae^{3x}$

$d^2y/dx^2 = 9Ae^{3x}$

$$9Ae^{3x} - 4(Ae^{3x}) = 10e^{3x}$$

$$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$$

$$Ae^{3x}(9-4) = 10e^{3x}$$

$$A = \frac{10e^{3x}}{5e^{3x}} = 2$$

$PI \Rightarrow y = 2e^{3x}$

General solution beomes

$$y = C\cosh 2x + D\sinh 2x + 2e^{3x}$$

3) $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = e^{-2x}$

$$\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1)$$

$$m_1 = m_2 = -1$$

$$y = e^{-x}(A + Bx) \Longrightarrow y$$

$$y = Ce^{-2x}$$

$$dy/dx = -2Ce^{-2x}$$

$$d^2y/dx^2 = 4Ce^{-2x}$$

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$Ce^{-2x}[4-4+1] = e^{-2x}$$

$$C = 1$$

$PI \Rightarrow y = e^{-2x}$

General Solution becomes

$$y = e^{-x}(A + Bx) + e^{-2x}$$

4) $d^2y/dx^2 + 25y = 5x^2 + x$

$d^2y/dx^2 + 25y = 0$

$m^2 + 25 = 0$

$m = \pm\sqrt{-1}\sqrt{25}$.

$y = C\cos 5x + D\sin 5x \Rightarrow C_f$

$y = Cx^2 + Dx + E$

$dy/dx = 2Cx + D$

$d^2y/dx^2 = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 + 25Dx + 25E + 2C = 5x^2 + x$

$25C = 5$

$\therefore C = 1/5$

$25D = 1 \quad \therefore D = 1/25$

$25E + 25C = 0$

$25(1/5) = -25E$

$E = 2/5 + -25 = -2/125$

$\therefore \quad y = 1/5x^2 + 1/25x - 2/125 \quad (P \cdot I)$

General solution becomes

$y = C\cos 5x + D\sin 5x + 1/5x^2 + 1/25x - 2/125$

5) $d^2y/dx^2 - 2\,dy/dx + y = 4\sin x$

$d^2y/dx^2 - 2\,dy/dx + y = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$\therefore m_1 = m_2 = 1$

$y = e^x(A + Bx) \longrightarrow C_f$

$y = C\cos x + D\sin x$

$dy/dx = -C\sin x + D\cos x$

$d^2y/dx^2 = -C\cos x - D\sin x$

$(-C\cos x - D\sin x) - 2(-C\sin x + D\cos x) + (C\cos x + D\sin x = 4\sin x$

$-D\sin x + 2C\sin x + D\sin x - C\cos x - 2D\cos x + C\cos x = 4\sin x$

Comparing coefficients

$-D + 2C + D = 4$

$\therefore 2C = 4 \therefore C = 2$

$\cancel{C} - 2D + \cancel{C} = 0$

$-2D = 0 \quad \therefore D = 0$

PI $\Rightarrow$ $y = 2\cos x + 0\sin x = 2\cos x$

General solution becomes

$y = e^{x}(A + Bx) + 2\cos x$

6) $d^2y/dx^2 + 4\,dy/dx + 5y = 2e^{-2x}$

$d^2y/dx^2 + 4\,dy/dx + 5y = 0$

$m^2 + 4m + 5 = 0$

$\therefore m = -2 + j$ or $m = -2 - j$

$y = e^{-2x}(C\cos x + D\sin x) \longrightarrow cf$

$y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$

$d^2y/dx^2 = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x}(4 - 8 + 5) = 2e^{-2x}$

$C = \dfrac{2e^{-2x}}{e^{-2x}} = 2$

PI $\Rightarrow$ $y = 2e^{-2x}$

General Solution becomes

$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$

at $x = 0$, $y = 1$ $\quad dy/dx = -2$

$1 = e^{0}(C\cos 0 + D\sin 0) + 2e^{0}$

$1 = C + 2 \therefore C = -1$

$dy/dx = e^{-2x}[-C\sin x + D\cos x] + [C\cos x + D\sin x] \cdot -2e^{-2x} - 4e^{-2x}$

at $x = 0$, $y = 1$, $dy/dx = -2$

$-2 = e^0[-C\sin 0 + D\cos 0] + [C\cos 0 + D\sin 0] \times -2e^0 - 4e^0$

$-2 = D - 2C - 4$

$C = -1$

$-2 = D - 2(-1) - 4$

$-2 + 2 = D$

$D = 0$

P.S $\Rightarrow$ $y = e^{-2x}[-\cos x] + 2e^{-2x}$

$\therefore y = -e^{-2x}\cos x + 2e^{-2x}$

7). $3\, d^2y/dx^2 - 2\, dy/dx - y = 2x - 3$

$3\, d^2y/dx^2 - 2\, dy/dx - y = 0$

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(3m+1)(m-1) = 0$

$m = -1/3$ or $m = 1$

$y = Ae^{-1/3x} + Be^{x} \Longrightarrow cf$

$y = Cx + D$

$dy/dx = C$

$d^2y/dx^2 = 0$

$0 - 2C - [Cx + D] = 2x - 3$

$-2C - Cx - D = 2x - 3$

Comparing Coefficients

$-C = 2$

$C = -2$

$-2C - D = -3$

$-2(2) - D = 3$

$4 - D = -3$

$D = 7$

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$y = -2x + 7$

General solution becomes

$$y = Ae^{-1/3x} + Be^{x} - 2x + 7$$

8) $d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{4x}$

soln

$d^2y/dx^2 - 6\,dy/dx + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 2m - 4m + 8 = 0$

$m(m-2) - 4(m-2) = 0$

$(m-4)(m-2) = 0$

$\therefore m = 4$ or $m = 2$

$y = Ae^{4x} + Be^{2x} \longrightarrow cf$

$y = Cxe^{4x}$

$dy/dx = C[x \cdot 4e^{4x} + e^{4x}]$

$= 4Cxe^{4x} + Ce^{4x}$

$d^2y/dx^2 = 16Cxe^{4x} + 4Ce^{x} + 4Ce^{4x}$

$16Cxe^{4x} + 8Ce^{4x} - C[4Cxe^{4x} + Ce^{4x}] + 8[Cxe^{4x}] = 8e^{4x}$

$16Cxe^{4x} + 8Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$Cxe^{4x}[16 - 24 + 8] + Ce^{4x}(8-6) = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$C = \dfrac{8e^{4x}}{2e^{4x}} = 4$

$y = 4xe^{4x}$

General Solution becomes

$$y = Ae^{4x} + Be^{2x} + 4xe^{4x}$$