

MUNIR MUSLEEMA M.

CHEMICAL ENGINEERING

14/ENGG01020

CHE 631

Assignment

Given that $y(0) = 5$ and $y'(0) = 7$

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 2e^{3t}$$

$$y'' - 3y' + 2y = 2e^{3t}$$

$\mathcal{L}\{f(t)\}$

$$s^2y(s) - sy(0) - y'(0) - 3[sy(s) - y(0)] + 2y(s) = \frac{2}{s-3}$$

$$s^2y(s) - 5s - 7 - 3[sy(s) - 5] + 2y(s) = \frac{2}{s-3}$$

$$s^2y(s) - 5s - 7 - [3sy(s) - 15] + 2y(s) = \frac{2}{s-3}$$

$$s^2y(s) - 5s - 7 - 3sy(s) + 15 + 2y(s) = \frac{2}{s-3}$$

$$s^2y(s) - 3sy(s) + 2y(s) - 5s - 8 = \frac{2}{s-3}$$

$$y(s) [s^2 - 3s + 2] - 5s - 8 = \frac{2}{s-3}$$

$$y(s) [s^2 - 3s + 2] = \frac{2}{s-3} + 5s + 8$$

$$y(s) [s^2 - s - 2s + 2] = \frac{2 + 5s(s-3) - 8(s-3)}{s-3}$$

$$y(s) [s(s-1) - 2(s-1)] = \frac{2 + 5s^2 - 15s - 8s + 24}{s-3}$$

$$y(s) [(s-1)(s-2)] = \frac{5s^2 - 23s + 26}{s-3}$$

$$y(s) = \frac{5s^2 - 23s + 26}{(s-1)(s-2)(s-3)}$$

$$y(s) = \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s-3}$$

$$= \frac{A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)}{(s-1)(s-2)(s-3)}$$

$$5s^2 - 23s + 26 = A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)$$

$$\text{Let } s = 1$$

$$5(1)^2 - 23(1) + 26 = A(1-2)(1-3) + B(1-1)(1-3) + C(1-1)(1-2)$$

$$5 - 23 + 26 = A(-1)(-2) + B(0)(-2) + C(0)(-1)$$

$$8 = 2A + 0 + 0$$

$$A = \frac{8}{2} = 4$$

$$\text{Let } s = 2$$

$$5(2)^2 - 23(2) + 26 = A(2-2)(2-3) + B(2-1)(2-3) + C(2-1)(2-2)$$

$$5 \times 4 - 46 + 26 = A(0)(-1) + B(1)(-1) + C(1)(0)$$

$$20 - 46 = 0 - B + 0$$

$$0 = -B$$

$$B = 0$$

$$\text{Let } s = 3$$

$$5(3)^2 - 23(3) + 26 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-1)(3-2)$$

$$5 \times 9 - 69 + 26 = A(1)(0) + B(2)(0) + C(2)(1)$$

$$45 - 69 = 0 + 0 + 2C$$

$$2 = 2C$$

$$C = 1$$

$$A = 4, B = 0, C = 1 \therefore y(s) = \frac{4}{s-1} + \frac{0}{s-2} + \frac{1}{s-3}$$

$$= \frac{4}{s-1} + \frac{1}{s-3}$$

$$y(t) = \mathcal{L}^{-1}\{y(s)\} = \mathcal{L}^{-1}\left[\frac{4}{s-1}\right] + \mathcal{L}^{-1}\left[\frac{1}{s-3}\right]$$
$$= 4e^t + e^{3t}$$