AE/ENG07/027

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1 $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Solve for L.H.S = 0 (C.F)

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2) = 0$

$m = -1 \quad m = 2$

$y = Ae^{\cancel{-x}} + Be^{2x}$

$y = Ae^{-x} + Be^{2x}$

Solve for RHS (PI)

$f(x) = 8$

$y = c$

$y' = 0 \quad y'' = 0$

Substituting

$0 - 0 - 2c = 8$

$-2c = 8$

$c = -4 \therefore PI = -4$

$GS = CF + PI$

$y = Ae^{-x} + Be^{2x} - 4$

(2) $d^2y/dx^2 - 4y = 10e^{3x}$

$GS = CF + PI$

CF : solve the LHS, $k^2 - 4 = 0$

$k^2 = 4 \therefore k = \pm 2$

$\therefore y = A\cosh 2x + B\sinh 2x$

PI, $f(x) = 10e^{3x}$, Assume $y = Ce^{3x}$

$y' = 3ce^{3x} \quad y'' = 9ce^{3x}$

Substitute into the given equation

$9ce^{3x} - 4(ce^{3x}) = 10e^{3x}$

$(9c - 4c)e^{3x} = 10e^{3x}$

$\frac{5c}{5} = \frac{10}{5} \therefore c = 2$

$PI = 2e^{3x}$

2) GS = CF + PI $\therefore$ $y = A\cosh 2x + B\sinh 2x + 2e^{3x}$

3) $d^2y/dx^2 + 2dy/dx + y = e^{-2x}$

$y'' + 2y' + y = 0$

$k^2e^{kx} + 2ke^{kx} + e^{kx} = 0$

$k^2 + 2k + 1 = 0$

$(k^2 + k)(k + 1) = 0$

$k(k+1)\ 1(k+1) = 0$

$k = -1$ twice

$CF = e^{-x}(A + Bx)$

PI, $f(x) = e^{-2x}$, $y = ce^{-2x}$

$y' = -2ce^{-2x}$, $y'' = 4ce^{-2x}$

substitute into the given equation

$4ce^{-2x} + 2(-2ce^{-2x}) + ce^{-2x} = e^{-2x}$

$4ce^{-2x} - 4ce^{-2x} + ce^{-2x} = e^{-2x}$

$ce^{-2x} = e^{-2x}$

$c = 1$

The PI $y = 1ce^{-2x}$ $\therefore$ $PI = e^{-2x}$

Recall GS = CF + PI

$\therefore$ $y = e^{-x}(A + Bx) + e^{-2x}$

4) $d^2y/dx^2 + 25y = 5x^2 + x$

CF: solve for the LHS, $k^2 + 25 = 0$

$k^2 = -25$ $\therefore$ $k = \pm 5j$

$\therefore$ $y = A\cos 5x + B\sin 5x$

PI: $f(x) = 5x^2 + x$, Assume $y = cx^2 + dx + E$

$y' = 2cx + d$, $y'' = 2c$

Substitute into the equation

$2c + 25(cx^2 + dx + E) = 5x^2 + x$

$2c + 25cx^2 + 25dx + 25E = 5x^2 + x$

$25cx^2 + 25dx + 2c + 25E = 5x^2 + x$

$\cancel{x}$ x^2: $\frac{25c}{25} = \frac{5}{25}$ $\quad$ x: $25d = 1$ $\quad$: $2c + 25E = 0$

$2(1/5) + 25E = 0$

$c = 1/5$, GS = CF + PI $\quad$ $\frac{2/5}{-25} = \frac{-25E}{-25}$

$\therefore$ $y = A\cos 5x + B\sin 5x$ ... $\quad$ $E = -2/125$

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4) $y = A\cos 5x + B\sin 5x + 1/5 x^2 + 4/25 x + -2/125$

5) $d^2y/dx^2 - 2dy/dx + y = 4\sin x$

$y'' - 2y' + y = 0$

$k^2 e^{kx} - 2ke^{kx} + e^{kx} = 0$

$k^2 - 2k + 1 = 0$

$k = 1$ twice

$CF = e^x (A + Bx)$

PI: $f(x) = 4\sin x \therefore y = C\cos x + D\sin x$

$y' = -C\sin x + D\cos x$

$y'' = -C\cos x - D\sin x$

Sub. into the given equations

$(-C\cos x + D\sin x) - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$(-C\cos x + C\cos x - 2D\cos x) + (-D\sin x + D\sin x + 2C\sin x) = 4\sin x$

$(-2D\cos x) + (2C\sin x) = 4\sin x$

$\frac{2C}{2} = \frac{4}{2}$

$C = 2$

$-2D\cos x = 0$

$\frac{-2D}{-2} = \frac{0}{-2}$ $D = 0$

$y = 2\cos x + 0\sin x$

$y = e^x (A + Bx) + 2\cos x$

6) $d^2y/dx^2 + 4dy/dx + 5y = 2e^{-2x}$

CF = Solve LHS = 0, $y'' + 4y' + 5y = 0$

$k^2 + 4k + 5 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - (4\times1\times5)}}{2\times1}$

$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2j}{2} = -2 \pm j$

$y = e^{-2x}(A\cos x + B\sin x)$

6) PI $\Rightarrow F(x) = 2e^{-2x}$, Assume $y = Ce^{-2x}$
$dy/dx = -2Ce^{-2x}$, $d^2y/dx^2 = 4Ce^{-2x}$
Sub into given equation
$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$
$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$
e^{-2x}: $4C - 8C + 5C = 2$
$C = 2$
$y = 2e^{-2x}$
(a S) $y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$
$x = 0$, $y = 1$
$1 = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-2(0)}$
$1 = 1(A + 0) + 2$
$1 = A + 2$
$A = -1$
$y = e^{-2x}(-\cos x + B\sin x) + 2e^{-2x}$
$dy/dx = e^{-2x}(\sin x + B\cos x) - 2e^{-2x}(-\cos x + B\sin x) - 4e^{-2x}$
If $x = 0$ and $dy/dx = -2$
$-2 = e^{-2(0)}(\sin(0) + B(\cos(0))) - 2e^{-2(0)}(-\cos(0) + B\sin(0))$
$- 4e^{-2(0)}$
$-2 = 1(0 + B) - 2(-1 + 0) - 4$
$-2 = B + 2 - 4$
$B = 0$
Particular Solution is
$y = e^{-2x}(-\cos x) + 2e^{-2x}$
$y = -e^{-2x}\cos x + 2e^{-2x}$
$y = e^{-2x}(2 - \cos x)$

7) $3\,d^2y/dx^2 - 2\,dy/dx - y = 2x-3$

$3y'' - 2y' - y = 0$

$3k^2 - 2k - 1 = 0$

$(3k^2 - 3k)(-1+k) = 0$

$3k(k-1) + 1(-1+k) = 0$

$(k-1)(3k+1) = 0$

$k_1 = 1, \quad k_2 = 1/3$

$Cf = Ae^x + Be^{-1/3x}$

PI; $f(x) = 2x-3$, Assume $y = (x+1)$

$y' = c, \quad y'' = 0$

Substitute into the given equation

$3(0) - 2(c) - (cx + b) = 2x - 3$

$-2c - cx + b = 2x - 3$

$-2c - cx + b = 2x - 3$

$(-2c + b) - cx = 2x - 3$ $\qquad \therefore\ Gs = Ae^x + Be^{-1/3x} - 2x + 7$

$-c = 2 \therefore c = -2$

$-2c + b = -3$ $\qquad \therefore 4 + b = -3$

$-2(-2) + b = 3$ $\qquad b = -3 - 4 \therefore b = 7$

8) $d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{4x}$

$y'' - 6y' + 8y = 0$

$k^2 - 6k + 8 = 0$

$k_1 = 2, \quad k_2 = 4$

$Cf = Ae^{2x} + Be^{4x}$

PI, $f(x) = 8e^{4x}$, Assume $y = cxe^{4x}$

$y' = c[x(4e^{4x}) + e^{4x}(1)]$

$y' = c[4xe^{4x} + e^{4x}] = 4cxe^{4x} + ce^{4x}$

$y'' = c[16xe^{4x} + 4e^{4x} + 4e^{4x}]$

$y'' = c[16xe^{4x} + 8e^{4x}]$

$= 16cxe^{4x} + 8ce^{4x}$

Sub into the given equation

$(16cxe^{4x} + 8ce^{4x}) - 6(4cxe^{4x} - ce^{4x}) + 8cxe^{4x} = 8e^{4x}$

$16cxe^{4x} + 8ce^{4x} - 24cxe^{4x} - 6ce^{4x} + 8cxe^{4x} = 8e^{4x}$

$8ce^{4x} - 6ce^{4x} = 8e^{4x}$

$2ce^{4x} = 8e^{4x}$

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$2c = 8 \quad \therefore \; c = 4$

$PI = 4x e^{4x}$

$GS = CF + PI$

$y = Ae^{2x} + Be^{4x} + 4x e^{4x}$