Solve the following

(1) $d^2y/dx^2 - dy/dx - 2y = 8$

(2) $d^2y/dx^2 - 4y = 10e^{3x}$

(3) $d^2y/dx^2 + 2dy/dx + y = e^{-2x}$

(4) $d^2y/dx^2 + 25y = 5x^2 + x$

(5) $d^2y/dx^2 - 2dy/dx + y = 4\sin x$

(6) $d^2y/dx^2 + 4\,dy/dx + 8y = 2e^{-2x}$, given that $x=0, y=1$ and $dy/dx = -2$

(7) $3\,d^2y/dx^2 - 2\,dy/dx - y = 2x - 3$

(8) $d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{4x}$

Solution

(1) $d^2y/dx^2 - dy/dx - 2y = 8$

$y'' - y' - 2y = 0$

$k^2e^{kx} - ke^{kx} - 2e^{kx} = 0$

$k^2 - k - 2 = 0$

$(k+1)(k-2) = 0 \quad \therefore k_1 = -1,\ k_2 = 2$

Complementary function, $y = Ae^{-x} + Be^{2x}$

PI, $f(x) = 8$ ie, A Constant, Assume $y = c$

$y' = 0, \quad y'' = 0$

Substituting the given equation

$0 - 0 - 2c = 8$

$-2c = 8 \quad \therefore c = -4$

PI is $y = -4$

General solution is $y = CF + PI$

$y = Ae^{-x} + Be^{2x} - 4$

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(2) $d^2y/dx^2 - 4y = 10e^{3x}$

GS = CF + PI

CF : Solve the LHS , $k^2 - 4 = 0$

$k^2 = 4 \quad \therefore k = \pm 2$

$\therefore \quad y = A\cosh 2x + B\sinh 2x$

PI, $f(x) = 10e^{3x}$, Assume $y = ce^{3x}$

$y' = 3ce^{3x} \quad y'' = 9ce^{3x}$

Substitute into the given equation

$9ce^{3x} - 4(ce^{3x}) = 10e^{3x}$

$(9c - 4c)\cancel{e^{3x}} = 10\cancel{e^{3x}}$

$\frac{5c}{5} = \frac{10}{5} \quad \therefore c = 2$

PI $= 2e^{3x}$

GS = CF + PI $\quad \therefore \quad y = A\cosh 2x + B\sinh 2x + 2e^{3x}$

(3) $d^2y/dx^2 + 2\,dy/dx + y = e^{-2x}$

$y'' + 2y' + y = 0$

$k^2e^{kx} + 2ke^{kx} + e^{kx} = 0$

$k^2 + 2k + 1 = 0$

$(k^2 + k)(k + 1) = 0$

$k(k+1) + (k+1) = 0$

$k = -1$ twice

CF $= e^{-x}(A + Bx)$

PI, $f(x) = e^{-2x}$, $y = ce^{-2x}$

$y' = -2ce^{-2x}$, $y'' = 4ce^{-2x}$

substitute into the given equation

$4ce^{-2x} + 2(-2ce^{-2x}) + ce^{-2x} = e^{-2x}$

$4ce^{-2x} - 4ce^{-2x} + ce^{-2x} = e^{-2x}$

$ce^{-2x} = e^{-2x}$

$c = 1$

The PI $y = 1ce^{-2x} \quad \therefore$ PI $= e^{-2x}$

GS = CF + PI

$\therefore y = \underline{e^{-x}(A + Bx) + e^{-2x}}$

(4) $d^2y/dx^2 + 25y = 5x^2 + x$

C_F: solve for the left hand side, $k^2 + 25 = 0$

$k^2 = -25$ $\therefore$ $k = \pm 5j$

$\therefore$ $y = A\cos 5x + B\sin 5x$

PI: $f(x) = 5x^2 + x$, Assume $y = Cx^2 + Dx + E$

$y' = 2Cx + D$, $y'' = 2C$

Substitute into the equation

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 + 25Dx + 2C + 2E = 5x^2 + x$

x^2: $\frac{25C}{25} = \frac{5}{25}$ $\quad$ x: $25D = 1$ $\quad$: $2C + 25E = 0$

$C = 1/5$ $\quad$ $D = 1/25$ $\quad$ $2(1/5) + 25E = 0$

$\frac{2/5}{-25} = \frac{-25E}{-25}$

$E = -2/125$

$GS = C_F + PI$

$y = A\cos 5x + B\sin 5x + 1/5 x^2 + 1/25 x + -2/125$

(5) $d^2y/dx^2 - 2dy/dx + y = 4\sin x$

$y'' - 2y' + y = 0$

$k^2 e^{kx} - 2ke^{kx} + e^{kx} = 0$

$k^2 - 2k + 1 = 0$

$k = 1$ twice.

$C_F = e^x(A + Bx)$

PI: $f(x) = 4\sin x$ $\therefore$ $y = C\cos x + D\sin x$

$y' = -C\sin x + D\cos x$

$y'' = -C\cos x - D\sin x$

Substitute into the given equation

$(-C\cos x + D\sin x) - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$(-C\cos x + C\cos x - 2D\cos x) + (-D\sin x + D\sin x + 2C\sin x) = 4\sin x$

$(-2D\cos x) + (2C\sin x) = 4\sin x$

$2C\sin x = 4\sin x$

$\frac{2C}{2} = \frac{4}{2}$

$C = 2$

$-2D\cos x = 0$

$\frac{-2D}{-2} = \frac{0}{-2}$ $D = 0$

$y = 2\cos x + 0\sin x$

$y = \underline{e^x(A + Bx) + 2\cos x}$

(7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3y'' - 2y' - y = 0$

$3k^2 - 2k - 1 = 0$

$(3k^2 - 3k)(-1 + k) = 0$

$3k(k-1) + 1(-1 + k) = 0$

$(k-1)(3k+1) = 0$

$k_1 = 1, \quad k_2 = -\frac{1}{3}$

$y = Ae^x + Be^{-\frac{1}{3}x}$

PI; $f(x) = 2x - 3$, Assume $y = Cx + D$

$y' = C, \quad y'' = 0$

Substitute into the given equation

$3(0) - 2(C) - (Cx + D) = 2x - 3$

$-2C - Cx + D = 2x - 3$ $\therefore GS = \underline{Ae^x + Be^{-\frac{1}{3}x} - 2x - 7}$

$(-2C + D) - Cx = 2x - 3$

$-C = 2 \quad \therefore C = -2$

$-2C + D = -3 \quad \therefore 4 + D = -3$

$-2(-2) + D = -3 \quad D = -3 - 4 \quad \therefore \underline{D = -7}$

6) $d^2y/dx^2 + 4\,dy/dx + 5y = 2e^{-2x}$

$\text{Q} =$ Solve LHS $= 0$, $y'' + 4y' + 5y = 0$

$k^2 + 4k + 5 = 0$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow \frac{-4 \pm \sqrt{4^2-(4\times1\times5)}}{2\times1}$$

$$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2j}{2} = -2 \pm j$$

$$y = e^{-2}(A\cos x + B\sin x)$$

PI $\Rightarrow f(x) = 2e^{-2x}$, Assume $y = ce^{-2x}$

$dy/dx = -2ce^{-2x}$, $d^2y/dx^2 = 4ce^{-2x}$

Substitute into the given equation

$4ce^{-2x} + 4(-2ce^{-2x}) + 5(ce^{-2x}) = 2e^{-2x}$

$4ce^{-2x} - 8ce^{-2x} + 5ce^{-2x} = 2e^{-2x}$

e^{-2x}: $4c - 8c + 5c = 2$

$c = 2$

$y = 2e^{-2x}$

(GS) $y = e^{2x}(A\cos x + B\sin x) + 2e^{-2x}$

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$1 = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-2(0)}$

$1 = 1(A + 0) + 2$

$1 = A + 2$

$A = -1$

$y = e^{-2x}(-\cos x + B\sin x) + 2e^{-2x}$

$\frac{dy}{dx} = e^{-2x}(\sin x + B\cos x) - 2e^{-2x}(-\cos x + B\sin x) - 4e^{-2x}$

if $x = 0$ and $\frac{dy}{dx} = -2$

$-2 = e^{-2(0)}(\sin(0) + B(\cos(0)) - 2e^{-2(0)}(-\cos(0) + B\sin(0)) - 4e^{-2(0)}$

$-2 = 1(0 + B) - 2(-1 + 0) - 4$

$-2 = B + 2 - 4$

$B = 0$

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Particular Solution is

$$y = e^{-2x}(-\cos x) + 2e^{-2x}$$

$$y = -e^{-2x}\cos x + 2e^{-2x}$$

$$y = e^{-2x}(2 - \cos x)$$

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(8) $d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{4x}$

$y'' - 6y' + 8y = 0$

$k^2 - 6k + 8 = 0$

$k_1 = 2,\ k_2 = 4$

$CF = Ae^{2x} + Be^{4x}$

PI, $f(x) = 8e^{4x}$, Assume $y = cxe^{4x}$

$y' = c[x(4e^{4x}) + e^{4x}(1)]$

$y' = c[4xe^{4x} + \cancel{4}e^{4x}] = 4cxe^{4x} + ce^{4x}$

$y'' = c[16xe^{4x} + 4e^{4x} + 4e^{4x}]$

$y'' = c[16xe^{4x} + 8e^{4x}]$

$= 16cxe^{4x} + 8ce^{4x}$

Substitute into the given equation

$(16cxe^{4x} + 8ce^{4x}) - 6(4cxe^{4x} + ce^{4x}) + 8cxe^{4x} = 8e^{4x}$

$16cxe^{4x} + 8ce^{4x} - 24cxe^{4x} - 6ce^{4x} + 8cxe^{4x} = 8e^{4x}$

$8ce^{4x} - 6ce^{4x} = 8e^{4x}$

$2c\cancel{e^{4x}} = 8\cancel{e^{4x}}$

$2c = 8 \quad \therefore c = 4$

$PI = 4xe^{4x}$

$GS = CF + PI$

$y = Ae^{2x} + Be^{4x} + 4xe^{4x}$

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