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16/SC104/002

Assignment

1.) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$, in homogenous form; $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

In auxillary form.

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2) = 0$

$m + 1 = 0$; $m - 2 = 0$

$m = -1$; $m_2 = 2$

CF:- $y = Ae^{-x} + Be^{2x}$

PI:- $y = c$

$\frac{dy}{dx} = 0$; $\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2(c) = 8$

$-2c = 8$

$c = \frac{8}{-2}$

$c = -4$

$GS = CF + PI$

$\therefore GS = y = Ae^{-x} + Be^{2x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

~~auxillary~~ homogenous form - $\frac{d^2y}{dx^2} - 4y = 0$

In auxillary form

$m^2 - 4 = 0$

$m^2 = 4$

$m = \sqrt{4}$

$m = \pm 2$

CF: ~~$y = \{C\cosh mx + D\sin 2x\}$~~

$y = C\cosh 2x + D\sin 2x$

P.I. $y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$\therefore 5C = 10$

$C = 10/5 = 2$

$GS = CF + PI$

GS: $y = C\cosh 2x + D\sin 2x + 2e^{3x}$

3.) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

In homogenous form

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

In auxillary form.

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m + 1 = 0$

$m = -1$

$CF = e^{-x}(A + Bx)$

$PI = y = Ce^{-2x}$

$\frac{dy}{dx} = -2e^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4ce^{-2x} + 2(-2(e^{-2x}) + ce^{-2x} = e^{-2x}$

$4ce^{-2x} - 4ce^{-2x} + ce^{-2x} = e^{-2x}$

$ce^{-2x} = e^{-2x}$

$c = \frac{e^{-2x}}{e^{-2x}}$

$c = 1$

G.S $= e^{-x}(A + Bx) + e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

In homogenous form m

$m^2 + 25 = 0$

$m^2 = -25$

$m = \sqrt{-25}$

$m = \sqrt{-1} \times \sqrt{25}$

$m = \pm j5$

Cf: $y = C\cos 5x + D\sin 5x$

PI: $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$2C + 25E + 25Dx + 25Cx^2 = 5x^2 + x$

$2C + 25E = 0$

$25D = 1 \therefore D = 1/25$

$25C = 5$

$C = \frac{5}{25}$

$C = 1/5$

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$25E = -2/5$

$E = -2/5 \div 25$

$E = -2/5 \times \frac{1}{25}$

$E = -2/125$

PI $= y = 1/5x^2 + 1/25x - 2/125$.

GS $= C\cos 5x + D\sin 5x + 1/5x^2 + 1/25x - 2/125$

$= C\cos 5x + D\sin 5x + \frac{1}{25}(5x^2 + 5x - 2)$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

In homogenous form.

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

In auxillary form

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1) = 0$.

$m = 1$

C.F $= y = e^x(A + Bx)$

PI: $y = A\cos x + B\sin x$

$\frac{dy}{dx} = -A\sin x + B\cos x$

$\frac{d^2y}{dx^2} = -A\cos x - B\sin x$

$-A\cos x - B\sin x - 2(-A\sin x + B\cos x)$
$+ A\cos x + B\sin x = 4\sin x$.

$-\cancel{A\cos x} - \cancel{B\sin x} + 2A\sin x - 2B\cos x$
$+ \cancel{A\cos x} + \cancel{B\sin x} = 4\sin x$

$2A\sin x - 2B\cos x = 4\sin x$

$2A = 4$

$A = 4/2 = 2$

$-2B = 0$

$B = \frac{0}{2} = 0$

PI: $y = 2\cos x$

GS: $y = e^{x}(A + Bx) + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm \sqrt{4}}{2}$

$m = \frac{-4 \pm \sqrt{-1} \times \sqrt{4}}{2}$

$m = \frac{-4 + j2}{2}$

$m = -2 + j$

$\beta = 1, \alpha = -2$

$y = e^{-2x}(A\cos x + B\sin x)$

PI: $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-3x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-3x}) + 5(Ce^{-3x}) =$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 5C = 2$

$C = 2$

PI: $y = 2e^{-3x}$

GS $= y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$\frac{dy}{dx} = Vdu + Udv$; $u = e^{-2x}$, $v = A\cos x + B\sin x$

$\frac{dy}{dx} = -2e^{-2x}(A\cos x + B\sin x) + e^{-2x}(-A\sin x + B\cos x) - 4e^{-2x}$

Since $\frac{dy}{dx} = -2$, $x = 0$, $y = 1$

$1 = e^{-2(0)}(A\cos 0 + B\sin 0) + 2e^{-2(0)}$

$1 = 1(A + 0) + 2$

$1 = A + 2$

$A = 1 - 2$

$A = -1$

$-2 = -2e^{-2(0)}(A\cos 0 + B\sin 0) + e^{-3(0)}(-A\sin 0 + B\cos 0) - 4e^{-2(0)}$

$-2 = -2(A + 0) + 1(0 + B) - 4$

$-2 = -2A + B - 4$

$-2A + B = 2$

$-2(-1) + B = 2$

$2 + B = 2$

$B = 2 - 2$

$B = 0$

PS: $y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-2x}$

$y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-2x}$

$y = e^{-2x} - \cos x + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

7.) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$3m + 1 = 0 \;;\; m - 1 = 0$

$m_1 = 1/3$

$(m-1)(3m+1) = 0$

$m_1 = 1 \quad , m_2 = -1/3$

$y = Ae^{x} + Be^{-1/3}$

$y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$3(0) - 2(C) - 1(Cx + D) = 2x - 3$

$-2C - Cx - D = 2x - 3$

$-2C - D - Cx = 2x - 3$

$-C = 2$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$4 + 3 = D$

$D = 7$

$PI = 2x + 7$

$G.S = Ae^{x} + Be^{-x/3} - 2x + 7$

8.) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{+x}$

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4) = 0$

$(m-4)(m-2) = 0$

$m - 4 = 0 \;,\; m - 2 = 0$

$m = 4 \;;\; m_2 = 2.$

$CF: y = Ae^{4x} + Be^{2x}$

$PI: y = Cxe^{4x}$

$y = uv \;,\; u = Cx \quad v = e^{4x}$

$\frac{dy}{dx} = v\,du + u\,dv$

$du = C$

$dv = 4e^{4x}$

$\frac{dy}{dx} = e^{4x}C + Cx \cdot 4e^{4x} = Ce^{4x} + 4Cxe^{4x}$

$\frac{d^2y}{dx^2} = 4Ce^{4x} + 4Ce^{4x} + 16Cxe^{4x} = 8Ce^{4x} + 16Cxe^{4x}$

$8Ce^{4x} + 16Cxe^{4x} - 6(Ce^{4x} + 4Cxe^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$8C + 16Cx - 6C - 24Cx + 8Cx = 8$

$8C - 6C = 8$

$2C = 8 \;,\; C = 8/2 \;,\; C = 4$

$PI: y = 4xe^{x}$

$GS = CF + PI$

$y = Ae^{4x} + Be^{2x} + 4xe^{x}$