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Chemical Engineering
Eng 331 - Engineering Maths III

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$, In homogenous form; $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

In auxillary form
$m^2 - m - 2 = 0$
$m^2 - 2m + m - 2 = 0$
$m(m-2) + 1(m-2) = 0$
$(m+1)(m-2) = 0$
$m+1 = 0$ $\quad m - 2 = 0$
$m_1 = -1$ $\quad m_2 = 2.$
CF: $y = Ae^{m_1 x} + Be^{m_2 x}$
$y = Ae^{-x} + Be^{2x}$
PI: $y = c$
$\frac{dy}{dx} = 0$ $\quad \frac{d^2y}{dx^2} = 0$
$0 - 0 - 2(c) = 8$
$-2c = 8$
$c = 8/-2$
$c = -4$
GS = CF + PI
PI: $y = -4$.
GS: $y = Ae^{-x} + Be^{2x} - 4$

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$, In homogenous form; $\frac{d^2y}{dx^2} - 4y = 0$

In auxillary form
$m^2 - 4 = 0$
$m^2 = 4$
$m = \sqrt{4}$
$m = \pm 2$
CF: $y = C\cosh mx + D\sin 2x$
$y = C\cosh 2x + D\sin 2x$

PI : $y = ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$\frac{5C}{5} = \frac{10}{5}$

$C = 2$

PI : $y = 2e^{3x}$

GS = CF + PI

$y = C\cos 2x + D\sin 2x + 2e^{3x}$

3. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

In homogenous form

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

In auxillary form

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m + 1 = 0$

$m = -1$

CF : $y = e^{mx}(A + Bx)$

$y = e^{-x}(A + Bx)$.

PI : $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = \frac{e^{-2x}}{e^{-2x}}$

$C = 1$

PI : $y = e^{-2x}$

GS = CF + PI

$y = e^{-x}(A+Bx) + e^{-2x}$

4. $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

In homogenous form

$\frac{d^2y}{dx^2} + 25y = 0$

In auxillary form

$m^2 + 25 = 0$

$m^2 = -25$

$m = \sqrt{25}$

$m = \sqrt{-1} = \sqrt{25}$

$m = \pm 5i$

CF : $y = C\cos 5x + D\sin 5x$

PI : $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$2C + 25E + 25Dx + 25Cx^2 = 5x^2 + x$

$2C + 25E = 0$

$25Dx = 1$

$25Cx^2 = 5x^2$

$C = \frac{5x^2}{25x^2}$

$C = 1/5$

$25D = 1$

$D = 1/25$

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$2/5 + 25E = 0$

$25E = -2/5$

$E = -2/5 \times 1/25$

$= -2/5 \times 1/25$

$= -2/125$

$= Cx^3 + Dx + E$

PI : $y = 1/5\,x^3 + 1/25\,x - 2/125$

GS : $y = y_{CF} + PI$

$y = (C\cos 5x + D\sin 5x + 1/5\,x^3 + 1/25\,x - 2/125$

$= C\cos 5x + D\sin 5x + \frac{1}{25}(25x^3 + 5x - 2)$

5 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

In homogeneous form

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

In auxiliary form

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1) = 0$

$m = 1$

CF : $y = e^x(A + Bx)$

PI : $y = A\cos x + B\sin x$

$\frac{dy}{dx} = -A\sin x + B\cos x$

$\frac{d^2y}{dx^2} = -A\cos x - B\sin x$

$-A\cos x - B\sin x - 2(-A\sin x + B\cos x) + A\cos x + B\sin x = 4\sin x$

$-A\cos x - B\sin x + 2A\sin x - 2B\cos x + A\cos x + B\sin x = 4\sin x$

$-A\cos x - 2B\cos x + A\cos x - B\sin x + 2A\cos x + B\sin x = 4\sin x$

$-2B\cos x + 2A\sin x = 4\sin x$

$-2B\sin x = 0$

$2A\sin x = 4\sin x$

$-2B = 0$

$B = 0/-2$

$B = 0$

$2A = 4$

$A = 4/2$

$A = 2$

PI : $y = 2\cos x + 0\sin x$

$y = 2\cos x$

GS = CF + PI

$y = e^{x}(A + Bx) + 2\cos x$

6 $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad a = 1,\ b = 4,\ c = 5$

$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm \sqrt{-1} \times \sqrt{4}}{2}$

$m = \frac{-4 \pm j2}{2}$

$m = -2 \pm j$

$\beta = 1,\ \alpha = -2$

$y = e^{-2x}(A\cos x + B\sin x)$

PI : $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 5C = 2$

$C = 2$

PI : $y = 2e^{-2x}$.

GS = CF + PI

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$\frac{dy}{dx} = u\,dv + v\,du$ where $u = e^{-2x}$ and $v = A\cos x + B\sin x$

$= -2e^{-2x}(A\cos x + B\sin x) + e^{-2x}(-A\sin x + B\cos x) - 4e^{-2x}$

Since $\frac{dy}{dx} = 2$, $x = 0$, $y = 1$

$1 = e^{-2(0)}(A\cos 0 + B\sin 0) + 2e^{-2(0)}$

$1 = 1(A + 0) + 2$

$1 = A + 2$

$A = 1 - 2$

$A = -1$

$-2 = -2e^{-2(0)}(A\cos 0 + B\sin 0) + e^{-2(0)}(-A\sin 0 + B\cos 0) - 4e^{-2(0)}$

$-2 = -2(A + 0) + 1(0 + B) - 4$

$-2 = -2A + B - 4$

$-2A + B = 2$

$-2(-1) + B = 2$

$2 + B = 2$

$B = 2 - 2$

$B = 0$

PS : $y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-2x}$

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

~~$= e^{-2x}(2 - \cos x)$~~

$y = e^{-2x} \cdot -\cos x + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$.

7 $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(m-1)(3m+1) = 0$

$m_1 = 1, \quad m_2 = -1/3$

$y = Ae^x + Be^{-x/3} \quad (CF)$

PI : $y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$3(0) - 2(C) - 1(Cx + D) = 2x - 3$

$-2C - Cx - D = 2x - 3$

$-2C - D - Cx = 2x - 3$

$-Cx = 2x$

$-C = 2$

$C = 2/-1$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$D = 4 + 3$

$D = 7$

PI : $y = -2x + 7$

GS = CF + PI

$y = Ae^x + Be^{-x/3} - 2x + 7$

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4)-2(m-4)=0$

$(m-4)(m-2)=0$

$m-4=0$ $\qquad$ $m-2=0$

$m_1=4$ $\qquad$ $m_2=2$

CF : $y = Ae^{4x} + Be^{2x}$

PI : $y = Cxe^{4x}$

$y = uv \quad , \quad u = Cx \quad v = e^{4x}$

$\frac{dy}{dx} = v\,du + u\,dv$

$du = C$

$dv = 4e^{4x}$

$\frac{dy}{dx} = e^{4x}\cdot C + Cx \cdot 4e^{4x} = Ce^{4x} + 4Cxe^{4x}$

$\frac{d^2y}{dx^2} = 4Ce^{4x} + 4Ce^{4x} + 16Cxe^{4x} = 8Ce^{4x} + 16Cxe^{4x}$

$8Ce^{4x} + 16Cxe^{4x} - 6(Ce^{4x} + 4Cxe^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$8Ce^{4x} + 16Cxe^{4x} - 6Ce^{4x} - 24Cxe^{4x} + 8Cxe^{4x} = 8e^{4x}$

$8C + 16Cx - 6C - 24Cx + 8Cx = 8$

$8C - 6C + 16Cx - 24Cx + 8Cx = 8$

$8C - 6C = 8$

$2C = 8$

$C = 8/2$

$C = 4$

PI : $y = Cxe^{x}$

$y = 4xe^{x}$

GS = CF + PI

$y = Ae^{4x} + Be^{2x} + 4xe^{x}$