Zakari Farida
15/ENG01/021
ENG 381

1) $d^2y/dx^2 - dy/dx - 2y = 8$, In homogenous form;
$d^2y/dx^2 - dy/dx - 2y = 0$
In auxiliary form
$m^2 - m - 2 = 0$
$m^2 - 2m + m - 2 = 0$
$m(m-2) + 1(m-2) = 0$
$(m+1)(m-2) = 0$
$m+1 = 0$
$m_1 = -1$ $\qquad$ $m-2 = 0$
$m_2 = 2$
CF: $y = Ae^{m_1 x} + Be^{m_2 x}$
$y = Ae^{-x} + B^{2x}$
P.I: $y = C$
$\frac{dy}{dx} = 0 \qquad \frac{d^2y}{dx^2} = 0$
$0 - 0 - 2(C) = 8$
$-2C = 8$
$C = 8/-2$
$C = -4$
$GS = CF + PI$
PI: $y = -4$
GS: $y = Ae^{-x} + Be^{2x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{2x}$, In homogenous form; $\frac{d^2y}{dx^2} - 4y = 0$
In auxiliary form
$m^2 - 4 = 0$
$m^2 = 4$
$m = \sqrt{4}$
$\cancel{m^2}$ $m = \pm 2$
CF: $y = C\cosh 2x + D\sinh 2x$
$y = C\cosh 2x + D\sinh 2x$

$PI: y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$\frac{5C}{5} = \frac{10}{5}$

$C = 2$

$PI: y = 2e^{3x}$

$GS = CF + PI$

$y = C\cosh 2x + D\sin 2x + 2e^{3x}$

(2) $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = e^{-2x}$

In homogenous form

$\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = 0$

In auxiliary form

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m+1 = 0$

$m = -1$

$CF: y = e^{mx}(A+Bx)$

$y = e^{-x}(A+Bx)$

$PI: y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4ce^{-2x} - 4ce^{-2x} + ce^{-2x} = e^{-2x}$

$ce^{-2x} = e^{-2x}$

$c = \frac{e^{-2x}}{e^{-2x}}$

$c = 1$

$PI : y = e^{-2x}$

$GS = CF + PI$

$y = e^{-x}(A + Bx) + e^{-2x}$

j) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

In homogenous form

$\frac{d^2y}{dx^2} + 25y = 0$

In auxillary form

$m^2 + 25 = 0$

$m^2 = -25$

$m = \sqrt{-25}$

$m = \sqrt{-1} \times \sqrt{25}$

$m = \pm j5$

$CF : y = C\cos 5x + D\sin 5x$

$PI : y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$2C + 25E + 25Dx + 25Cx^2 = 5x^2 + x$

$2C + 25E = 0$

$25D = 1 \therefore D = 1/25$

$25C = 5$

$C = 5/25$

$C = 1/5$

$2C + 25E = 0$

$2C(1/5) + 25E = 0$

$25E = -2/5$

$E = -2/5 \div 25$

$E = -2/5 \times 1/25$

$E = -2/125$

$PI = y = 1/5 x^2 + 1/25 x - 2/125$.

$GS = C\cos 5x + D\sin 5x + 1/5 x^2 + 1/25\, x - 2/125$

$= C\cos 5x + D\sin 5x + 1/125(25x^2 + 5x - 2)$

5) $d^2y/dx^2 - 2dy/dx + y = 4\sin x$, In homogenous form

$d^2y/dx^2 - 2dy/dx + y = 0$

In auxiliary form

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1) = 0$

$m = 1$

$C.F = y = e^x(A + Bx)$

$P.I : y = A\cos x + B\sin x$

$= -A\sin x + B\cos x$

$d^2y/dx^2 = -A\cos x - B\sin x$

$-A\cos x - B\sin x - 2(-A\sin x + B\cos x) + A\cos x + B\sin x =$

$4\sin x \quad -A\cos x - B\sin x + 2A\sin x - 2B\cos x$

$+ A\cos x + B\sin x = 4\sin x$

$2A\sin x - 2B\cos x = 4\sin x$

$2A = 4$

$A = 4/2 = 2$

$-2B = 0$

$B = 0/2 = 0$

$PI : y = \cos x$

$GS : y = e^x(A + Bx) + 2\cos x$

(c) $d^2y/dx^2 + 4dy/dx + 5y = 2e^{-2x}$

$d^2y/dx^2 + 4dy/dx + 5y = 0$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm \sqrt{4}}{2}$

$m = \frac{-4 \pm \sqrt{-1} \times \sqrt{4}}{2}$

$m = \frac{-4 \pm j2}{2}$

$m = -2 \pm 1$

$\beta = 1, \alpha = -2$

$y = e^{-2x}(A\cos x + B\sin x)$

PI : $y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$

$d^2y/dx^2 = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) + 2e^{-2x} + Ce^{-2x}$

$-8Ce^{-2x} + 5Ce^{-2x} \pm 2e^{-2x}$

$4C - 8C + 5C = 2$

$C = 2$

P.I : $y = 2e^{-2x}$

G.S : $y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$dy/dx = Vdu + Vdv$; $u = e^{-2x}$, $V = A\cos x + B\sin x$

$dy/dx = -2e^{-2x}(A\cos x + B\sin x) + e^{-2x}(-A\sin x + B\cos x)$

$-4e^{-2x}$

Since $dy/dx = -2$, $x = 0$, $y = 1$

$1 = e^{-2(0)}(A\cos x + B\sin x) + 2e^{-2(0)}$

$1 = 1(A + 0) + 2$

$1 = A + 2$

$A = 1 - 2$

$A = -1$

$-2 = -2e^{-2(0)}(A\cos 0 + B\sin 0) + e^{-2(0)}(-A\sin 0 + B\cos 0) - 4e^{-2(0)}$

$-2 = -2(A+0) + 1(0+B) - 4$

$-2 = -2A + B - 4$

$-2A + B = 2$

$-2(-1) + B = 2$

$2 + B = 2$

$B = 2 - 2$

$B = 2$

$PI: y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-2x}$

$y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-2x}$

$y = e^{-2x} - \cos x + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

7) $3d^2y/dx^2 - 2dy/dx - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

$3m(m-1)\ 1(m-1) = 0$

$3(m+1 = 0 \quad 1: m - 1 = 0$

$m_1 = 1/3$

$(m-1)\quad(3m+1) = 0$

$m_1 = 1 \quad m_2 = -1/3$

$y = Ae^{x} + Be^{-1/3}$

$y = Cx + D$

$dy/dx = C$

$d^2y/dx^2 = 0$

$3(0) - 2(C) - 1(Cx + D)\ 2x - 3$

$-2C - Cx - D = 2x - 3$

$-x - D - Cx = 2x - 3$

$-C = 2$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$4 + 3 = D$

$D = 7$

$P.I = 3x + 7$

$G.S = Ae^{x} + Be^{-x/3} - 2x + 7$

8) $d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{4x}$

$d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{4x}$

$d^2y/dx^2 - dy/dx + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4) = 0$

$m - 4 = 0 \quad m - 2 = 0$

$m = 4 \;:\; m_2 = 2$

$CF : y = Ae^{4x} + Be^{2x}$

$PI : y = Cxe^{4x}$

$y = uv,\ u = cx \quad v = e^{4x}$

$du = c$

$dv = 4e^{4x}$

$dy/dx = e^{4x}c + cx \cdot 4e^{4x} = Ce^{4x} + 4Cxe^{4x}$

$d^2y/dx^2 = 4Ce^{4x} + 4Ce^{4x} + 16Cxe^{4x} = 8Ce^{4x} + 16Cxe^{4x}$

$8Ce^{4x} + 16Cxe^{4x} - 6(Ce^{4x} + 4Cxe^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$8C + 16Cx - 6C - 24Cx + 8Cx = 8$

$8C - 6C = 8$

$2C = 8,\ C = 8/2,\ C = 4$

$PI : y = 4xe^{x}$

$G.S = CF + PI$

$y = Ae^{4x} + Be^{2x} + 4xe^{x}$