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Chemical Engineering

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$ — ①

When $f(x) = 0$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$(m^2 - 2m)(+m - 2) = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2)$

$m = 2$ or $m = -1$

CF $\Rightarrow y = Ae^{2x} + Be^{-x}$

To get PI

let $y = c$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

—① becomes

$-2c = 8$

$c = \frac{8}{-2}$

$c = -4$

PI, $y = -4$

GS = CF + PI

$y = Ae^{2x} + Be^{-x} - 4$ //

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$ — ①

$f(x) = 0$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m = \pm\sqrt{4}$

$m = +2$ or -2

$y = Ae^{2x} + Be^{-2x}$ — CF

To get PI

let $y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

—① becomes

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$Ce^{3x}(9-4) = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$C = 2$

PI, $= y = 2e^{3x}$

G.S = CF + PI

$y = Ae^{2x} + Be^{-2x} + 2e^{3x}$ //

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x} \quad -①$$

when $f(x) = 0$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$(m^2 + m)(m + 1) = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1$ twice

$y = e^{-x}(A + Bx) \quad -$ CF

To get PI

let $y = Ce^{-2x}$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$-①$ becomes

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x}(4 - 4 + 1) = e^{-2x}$

$C\cancel{e^{-2x}} = \cancel{e^{-2x}}$

$C = 1$

$PI = y = e^{-2x}$

$GS = CF + PI$

$y = e^{-x}(A + Bx) + e^{-2x}$ //

4) $$\frac{d^2y}{dx^2} + 25y = 5x^2 + x \quad -①$$

let $f(x) = 0$

$$\frac{d^2y}{dx^2} + 25y = 0$$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{-25}$

$m = \pm j5$

$y = C\cos 5x + D\sin 5x \quad -$ CF

To get PI

$y = Cx^2 + Dx + E$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$-①$ becomes

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25C = 5$

$C = 1/5$

$25Dx = 1.$

$D = \frac{1}{25}$

$2C + 25E = 0$

$2(\frac{1}{5}) + 25E = 0$

$\frac{2}{5} + 25E = 0$

$25E = -\frac{2}{5}$

$E = \frac{-2}{125}$

$$PI = y = \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$$

$$GS = C\cos 5x + D\sin 5x + \frac{x^3}{5} + \frac{x}{25} - \frac{2}{125} \quad //$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x \quad -①$$

$f(x) = 0$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$m^2 - 2m + y = 0$

$m^2 - m - m + 1 = 0$

$(m^2 - m)(-m + 1) = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1)$

$m_1 = 1$ twice

$y = e^x(A + Bx)$ — CF

To get PI

let $y = C\cos x + D\sin x$

$$\frac{dy}{dx} = -C\sin x + D\cos x$$

$$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$$

$-①$ becomes

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x)$
$+ C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x$
$+ C\cos x + D\sin x = 4\sin x$

$-C\cos x - 2D\cos x + C\cos x - D\sin x +$
$2C\sin x + D\sin x = 4\sin x$

$\cos x(-C - 2D + C) + \sin x(-D + 2C + D)$
$= 4\sin x$

$\cos x(-2D) + \sin x(2C) = 4\sin x$

$-2D = 0$

$D = 0$

$2C = 4$

$C = 2$

$PI = y = 2\cos x + 0\sin x$

$y = 2\cos x$

$G.S = PI + CF$

$y = e^x(A + Bx) + 2\cos x$ //

6) $$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x} \quad -①$$

$x = 0,\ y = 1$ and $\frac{dy}{dx} = -2$

$f(x) = 0$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$

$m^2 + 4m + 5 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-4 \pm \sqrt{16 - 4 \times 5}}{2}$$

$$\frac{-4 \pm 16 - 20}{2}$$

$$\frac{-4 \pm \sqrt{-4}}{2}$$

$$\frac{-4 \pm j\sqrt{4}}{2}$$

$-2 \pm j$, $y = e^{-2x}(C\cos x + D\sin x)$ — CF

To get PI

The general form of the RHS $= Ce^{-2x}$

but this term e^{-2x} is already contained in the CF so, assume

$y = Cxe^{-2x}$

$\frac{dy}{dx} = Cx(-2e^{-2x}) + e^{-2x}(C)$
$-2Cxe^{-2x} + Ce^{-2x}$

$\frac{d^2y}{dx^2} = -2Cx(-2e^{-2x}) + e^{-2x}(-2C) + -2Ce^{-2x}$

$= 4Cxe^{-2x} - 2Ce^{-2x} - 2Ce^{-2x}$
$= 4Cxe^{-2x} - 4Ce^{-2x}$

-① becomes
$4Cxe^{-2x} - 4Ce^{-2x} + 4(-2Cxe^{-2x} + Ce^{-2x})$
$+5(Cxe^{-2x}) = 2e^{-2x}$

$4Cxe^{-2x} - 4Ce^{-2x} - 8Cxe^{-2x} + 4Ce^{-2x}$
$+5Cxe^{-2x} = 2e^{-2x}$

$4Cxe^{-2x} - 8Cxe^{-2x} + 5Cxe^{-2x} - \cancel{4Ce^{-2x}} + \cancel{4Ce^{-2x}}$
$= 2e^{-2x}$

$\cancel{e^{-2x}}(4Cx - 8Cx + 5Cx) = 2\cancel{e^{-2x}}$
$Cx = 2$

PI, $y = 2e^{-2x}$
$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$ — GS
at $x=0$ and $y=1$
$1 = e^{-2(0)}(C\cos(0) + D\sin(0)) + 2e^{-2(0)}$
$1 = 1(C+0) + 2$
$1 = C+2$
$C = -1$

$\frac{dy}{dx} = e^{-2x}(-C\sin x + D\cos x) + (C\cos x + D\sin x) - 2e^{-2x} - 4e^{-2x}$.

at $x=0$, $y=1$ and $\frac{dy}{dx} = -2$

$-2 = D - 2C - 4$
$-2+4 = D - 2C$
$2 = D+2$
$D = 0$
$y = e^{-2x}(-\cos x + 0) + 2e^{-2x}$
$y = e^{-2x}(-\cos x x) + 2e^{-2x}$
$y = e^{-2x}(2 - \cos x)$ //

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$ —①

Let $f(x) = 0$
$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$

$3m^2 - 2m - 1 = 0$
$3m^2 - 3m + m - 1 = 0$
$(3m^2 - 3m) + (m-1) = 0$
$3m(m-1) + 1(m-1) = 0$
$(3m+1)(m-1) = 0$
$m = 1$, $m = -1/3$
$y = Ae^{x} + Be^{-1/3x}$
To find PI
Let $y = Cx + D$
$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$
-① becomes
$0 - 2C - Cx + D = 2x - 3$
$-Cx - 2C - D = 2x - 3$
$-Cx = 2x$
$-C = 2$
$C = -2$
$-2C - D = -3$
$+4 - D = -3$
$-D = -7$, $D = 7$, PI $= -2x + 7$
$y = Ae^{x} + Be^{-1/3x} - 2x + 7$ //

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$ — ①

$f(x) = 0$

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$(m^2 - 4m)(-2m + 8) = 0$

$m(m-4) - 2(m-4) = 0$

$(m-4)(m-2)$

$m = 4$ or 2

$y = Ae^{4x} + Be^{2x}$ — CF

To get PI

The general form of the RHS is Ce^{4x} but this term e^{4x} is already contained in the CF, so assume.

$y = Cxe^{4x}$

$\frac{dy}{dx} = Cx(4e^{4x}) + Ce^{4x}$

$= 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

— ① become

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$16Cxe^{4x} - 24Cxe^{4x} + 8Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$

$e^{4x}(0 + 2C) = 8e^{4x}$

$2C = 8$

$C = 4$

$y = 4xe^{4x}$

G.S

CF + PI

$Ae^{4x} + Be^{2x} + 4xe^{4x}$ //

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