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ENG 281
COMPUTER ENGINEERING
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ENG 281 ASSIGNMENT 2

1] $x = \cos t + t \sin t$

$y = \sin t - t \cos t$

a] An expression for the radius of curvature (R) in terms of t

Ans.

$x = \cos t + t \sin t$

$\frac{dx}{dt} = -\sin t + t\cos t + \sin t = t \cos t$

$y = \sin t - t \cos t$

$\frac{dy}{dt} = \cos t + t \sin t - \cos t = t \sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{t \sin t}{t \cos t}$

$\frac{dy}{dx} = \frac{\sin t}{\cos t}$

$\frac{d^2y}{dx^2} = ?$

$u = \sin t, \quad v = \cos t$

$\frac{du}{dt} = \cos t, \quad \frac{dv}{dt} = -\sin t$

$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{2}$

$$\frac{\cos t(\cos t) - (\sin t)(-\sin t)}{(\cos t)^2} \times \frac{1}{t\cos t}$$

$$\frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$$

$$\Rightarrow \frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{t\cos^3 t}$$

$$R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left(1 + \left(\frac{\sin t}{\cos t}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$$

$$R = \frac{\left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t}\cos^3 t}$$

$$R = \left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)^{3/2} \div \frac{1}{t\cos^3 t}$$

$$R = \left(\frac{1}{\cos^2 t}\right)^{3/2} \div \frac{1}{t\cos^3 t}$$

$$R = \frac{\frac{1}{3}\cos^3 t}{\frac{1}{t}\cos^3 t}$$

$$R = \frac{1}{\cos^3 t} \times t\cos^3 t$$

$$R = t$$

2] Expressions for the coordinates (h, k) of the centre of curvature in terms of t

Ans:

$$h = x_1 - R\sin t$$
$$k = y_1 + R\cos\theta$$

$R = t, \quad \theta = \tan^{-1}\left(\frac{dy}{dx}\right)$

$$\frac{dy}{dx} = \frac{\sin t}{\cos t}$$
$$= \tan t$$

$$\therefore \quad \theta = \frac{1}{\tan\theta} \times \tan t$$

$$\theta = t$$

$$h = x_1 - t\sin t$$
$$k = y_1 + t\cos t$$

but $x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$$h = \cos t + t\sin t - t\sin t$$
$$h = \cos t$$

$$k = \sin t - t\cos t + t\cos t$$
$$k = \sin t$$

$$(h, k) = (\cos t, \sin t)$$