Solutions QUESTION [1A]

1a] $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right]$

$= \left[\frac{\left(\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4}\right) \sin(\cos \pi/2)}{\pi/2 - \pi/2} \right]$

$= \left[\frac{\left(\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4}\right) \sin((\cos \pi/2)}{\pi/2 - \pi/2} \right] = \frac{\left(\frac{\pi^2}{4} - \frac{\pi}{4}\right) \sin\left(\cos \frac{\pi}{2}\right)}{0}$

ie indeterminant.

$\therefore$ using L'Hopital's Law, dy/dx of the numerator $= u\frac{dv}{dx} + v\frac{dy}{dx}$

$\frac{dy}{dx} =$ let $u = x^2 - \frac{\pi}{4}$

and $v = \sin(\cos x)$

$\frac{dy}{dx} = 2x \qquad \frac{dy}{dx} = ?$

$\frac{d}{dx} \sin(\cos x) =$ let $\cos x = \omega$

$v = \sin\omega$

$\frac{dv}{d\omega} = \cos\omega \quad , \quad \therefore \frac{d\omega}{dx} = -\sin x$

$\frac{dv}{dx} = \frac{dv}{d\omega} \times \frac{d\omega}{dx} = -\sin x \cos(\cos x)$

$= \frac{\left(x^2 - \frac{\pi}{4}\right) \times -\sin x \cos(\cos x) + \sin(\cos x)(2x)}{1}$

$= \left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \times -\sin 90 \cos(\cos 90) + \sin \cos 90 \times 2\left(\frac{\pi}{2}\right)$

$= \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right) \times -1 + 0 \times \pi$

$= -\frac{\pi^2}{4} + \frac{\pi}{4} \quad ; \quad = \frac{\pi}{4} - \frac{\pi^2}{4}$

$\therefore$

$\lim_{x = \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right] = \frac{\pi(1 - \pi)}{4}$

QUESTION 1B

1b] $\lim_{x\to \pi/2} \ln \left[\exp \frac{(3x^2+2x-1)}{x+1} \right]$

$= \lim_{x\to \frac{\pi}{2}} \ln \left(\exp \left[\frac{(3x-1)(x+1)}{x+1^2} \right] \right)$

$= \lim_{x\to \pi/2} \ln(\exp(3x-1))$

$= \lim_{x\to \pi/2} (3x-1) = 3(\pi/2)-1$

$= \frac{3\pi-1}{2} = \frac{3\pi}{2} - \frac{1}{1}$

$\therefore \lim_{x\to \pi/2} \ln \left[\exp \frac{(3x^2+2x-1)}{x+1} \right] = \frac{3\pi-2}{2}$

QUESTION 1C

1c] $\lim_{x\to 2+\sqrt{3}} \cos \left(\frac{\sin^{-1}(x-2)}{(x-\sqrt{3}} \right)$

$= \lim_{x\to 2+\sqrt{3}} \cos \left[\frac{\sin^{-1}(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3}} \right]$

$= \cos \left[\sin^{-1} \left[\frac{\sqrt{3}}{2} \right] \right]$

$= \cos[\sin^{-1}(0.8660)]$

$\Rightarrow \cos 60^\circ$

$= \frac{1}{2}$

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QUESTION 1D

b) $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

$\Rightarrow \lim_{x \to 4} \left[\frac{(x-4)(x-4)}{(x-4)(x-1)} \right]$

$= \lim_{x \to 4} \left[\frac{x-4}{x-1} \right]$

$= \frac{4-4}{4-1} = \frac{0}{3} = 0$

QUESTION 2A

$U_n = \frac{2}{(n+1)(n+2)}$

$U_{n+1} = \frac{2}{(n+2)(n+3)}$

ratio: $\frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$

$\frac{U_{n+1}}{U_n} = \frac{n+1}{n+3}$

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \lim_{n \to \infty} \frac{n+1}{n+3}$

$= \frac{\frac{n}{n} + \frac{1}{n}}{\frac{n}{n} + \frac{3}{n}} = \frac{1 + \frac{1}{n}}{1 + \frac{3}{n}} = \frac{1+0}{1+0} = \frac{1}{1} = 1$

Since $\lim_{n \to \infty} \frac{U_{n+1}}{U_n}$

$\therefore$ The series is inconclusive

QUESTION 2B

using the comparison test

recall;

$$\left[\frac{1}{1^r} + \frac{1}{2^r} + \frac{1}{3^r} + \frac{1}{4^r} + \cdots \frac{1}{n^r}\right] = \sum_{n-1}^{\infty} \frac{1}{n^p}$$

$$\Rightarrow \left[\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots \frac{2}{n^2}\right] = \sum_{n=1}^{\infty} \frac{2}{n^2}$$

$P = 2$

Since $P > 1$, the series converge

QUESTION 3

$$U_n = \frac{x^n}{(2n+1)^3}, \quad U_{n+1} = \frac{x^{n+1}}{(2n+2)^3}$$

$$\lim \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+2)^3} \times \frac{(2n+1)^3}{x^n}$$

$$\frac{x(2n+1)^3}{(2n+2)^3} = \frac{8n^3 + 12n^2 + n + 1}{8n^3 + 24n^2 + 24n + 8}$$

divide by n^3

$$\Rightarrow \frac{(8 + 2/n + 1/n^2 + 1/n^3)}{(8 + 12/n + 24/n^2 + 8/n^3)}$$

as $n \to \infty$

$1/n \to 0$

$$\frac{8x}{8} \geq x - 1$$

$x < 1$

QUESTION 4

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

by using L'Hopital's rule

$$y = \left(\frac{\sin x - \cos x}{x^3} \right)$$

$$\frac{dy}{dx} = \left[\frac{\cos x + \sin x}{3x^2} \right]$$

$$\frac{d^2y}{dx^2} = \frac{-\sin x + \cos x}{6x}$$

$$\frac{d^2y}{dx^3} = \frac{-\cos x - \sin x}{6}$$

$$\lim_{x \to 0} = \frac{-\cos 0 - \cos 0}{6} = \frac{-1-0}{6} = \frac{-1}{6}$$

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right] = -\frac{1}{6}$$