NAME: Ambaiowei Viktor
DEPT Mechanical Engineering
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1 Evaluate the following limits of function

a $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$ b) lim

c $\lim_{x \to 2\sqrt{3}} \cos \left\{ \frac{\sin^{-1}(x-2)}{x\sqrt{3}} \right\}$ d) $\lim_{x \to 4} \left\{ \frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right\}$

2 Determine whether each of the following Series is Convergent

a $\frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \frac{2}{5 \times 6}$

b $\frac{2}{1^3} + \frac{2}{2^3} + \frac{2}{3^3} + \frac{2}{4^3} \ldots$

c $U_n = \frac{1 + 2n^3}{1 + n^3}$

3 Find the range of values of x for which the series below is absolutely convergent;
$x/27 + x^2/125 + \ldots + \frac{x^n}{(2n+1)^3}$

4 Evaluate using L'Hopital Rule
$\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right)$

Sol

$$\lim_{x \to \pi/2} \left\{ \frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right\}$$

$dy/dx = \frac{(2x - 0)(-\sin^2 x)}{1 - 0}$, $y = \left[(x^2 - \pi/4) \right]$

$$y - \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$$

direct substituting $x \to \pi/2$

$= 2(\pi/2)(-\sin^2(\pi/2))$

$-\pi(-1)^2$

$= -\pi$

$$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right] = -\pi$$

b) $\lim_{x \to -1} \ln \left[\exp \left(\frac{3x^2 + 2x - 1}{x + 1} \right) \right]$

$\ln \left[\exp \left(\frac{(3x - 1)(x + 1)}{(x + 1)} \right) \right]$

$\ln \left[\exp [3(-1)(-1) \right]$

$\ln (\exp (-3 - 1) = -4$

c $\lim_{x \to 2 + \sqrt{3}} \cos \left[\sin^{-1} \left(\frac{x - 2}{x - \sqrt{3}} \right) \right]$

$\cos \left[\sin^{-1} \left(\frac{2 + \sqrt{3} - 2}{2 + \sqrt{3} - \sqrt{3}} \right) \right]$ $\cos \left[\sin^{-1} \frac{\sqrt{3}}{2} \right]$

$\cos 60 = 1/2$

d $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

$\lim_{x \to 4} \left[\frac{(x - 4)(x + 4)}{(x - 1)(x - 4)} \right]$

$\lim_{x \to 4} \left[\frac{x + 4}{x - 1} \right]$

$\frac{4 - 4}{4 - 1} = \frac{0}{3} = 0$

2) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} \dots \frac{2}{(n+1)(n+2)}$

$u_n = \frac{2}{(n+1)(n+2)}$, $u_{n+1} = \frac{2}{(n+2)(n+3)}$

$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2} = \frac{2(n^2+3n+2)}{2(n^2+5n+6)}$

$\frac{n^2/n^2 + 3n/n^2 + 2/n^2}{n^2/n^2 + 5n/n^2 + 6/n^2} = \frac{1+3/n+2/n^2}{1+5/n+6/n^2}$

$n\to\infty$, $\frac{1+0+0}{1+0+0} = \frac{1}{1} = 1$

$\frac{u_{n+1}}{u_n} = 1$, Series is divergent or convergent

Further test

$\lim_{n\to\infty} u_n = \frac{2}{(n+1)(n+2)} = \frac{2}{2} = 2/2 = 1$

$u_n \neq 0$ ∴ The Series is divergent

b) $\frac{2}{1^2} + \frac{2^2}{2^2} + \frac{2^3}{3^2} + \frac{2^4}{4^2} + \dots \frac{2^n}{n^2}$

$u_n = \frac{2^n}{n^2}$, $u_{n+1} = \frac{2^{n+1}}{(n+1)^2}$

$\lim_{n\to\infty} = \frac{u_{n+1}}{u_n} = \frac{2^{n+1}}{n^2+2n+1} \times \frac{n^2}{2^n} \to \frac{2^n \times 2 \times n^2}{2^n(n^2+2n+1)}$

$\lim_{n\to\infty} \frac{2n^2}{n^2+2n+1} = \frac{2n^2/n^2}{n^2/n^2 + 2n/n^2 + 1/n^2} = \frac{2}{1+2/n+1/n^2}$

$n\to\infty$, $\frac{2}{1+0+0} = 2$

$\frac{u_{n+1}}{u_n} > 1$ the Series is divergent

c) $u_n = \frac{1+2n^2}{1+n^2}$

$\lim_{n\to\infty} \frac{1+2n^2}{1+n^2} = \frac{1/n^2+2}{1/n^2+1} = \frac{2+0}{1+0} = \frac{2}{1} = 2$

$n \to 1/n \to 0$

$u_n \neq 0 \therefore$ series is divergent

3. $\frac{x}{1\cdot 2} + \frac{x^2}{1\cdot 2\cdot 3} + \cdots \frac{x^n}{(2n+1)^3}$

$u_n = \frac{x^n}{(2n+1)^3}$, $u_{n+1} = \frac{x^{n+1}}{(2n+2)^3}$

$\lim \frac{u_{n+1}}{u_n} = \frac{x^{n+1}}{(2n+2)^3} \times \frac{(2n+1)^3}{x^n}$

$\frac{x(2n+1)^3}{(2n+2)^3} = \frac{x[8n^3+12n^2+n+1]}{8n^3+24n^2+24n+8}$

divide by n^3

$\frac{x(8+12/n+1/n^2+7/n^3)}{(8+12/n+24/n^2+8/n^3)}$

as $n \to \infty$ $1/n \to 0$

$\frac{8x}{8} \quad \Sigma x = 1$

$x < 1$

4. $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right]$

by using L'hopitals rule

$y = \left[\frac{\sin x - \cos x}{x^3}\right]$

$dy/dx = \left[\frac{\cos x + \sin x}{3x^2}\right]$

$\frac{d^2y}{dx^2} = \frac{-\sin x + \cos x}{6x}$

$\frac{d^3y}{dx^3} = \frac{-\cos x - \sin x}{6}$

$\lim_{x \to 0} = \frac{-\cos 0 - \cos 0}{6} = \frac{-1-0}{6} = -\frac{1}{6}$

$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right] = -\frac{1}{6}$