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1 $x = \cos t + t \sin t$
 $y = \sin t - t \cos t$

9 An expression for the radius of curvature (R) in terms of t

Ans

$x = \cos t + t \sin t$

$\frac{dx}{dt} = -\sin t + t \cos t + \sin t = t \cos t$

$y = \sin t - t \cos t$

$\frac{dy}{dt} = \cos t + t \sin t - \cos t = t \sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{t \sin t}{t \cos t}$

$\frac{dy}{dx} = \frac{\sin t}{\cos t}$

$\frac{d^2y}{dx^2}$

$u = \sin t, v = \cos t$

$\frac{du}{dt} = \cos t$

$\frac{dv}{dt} = -\sin t$

$\frac{d^2y}{dx^2} = \frac{u \frac{dv}{dt} - v \frac{du}{dt}}{v^3}$

$\frac{\cos t (\cos t) - (\sin t) (-\sin t)}{(\cos t)^3} \times 1$

$\frac{\cos^2 t + \sin^2 t}{\cos^3 t} \times 1$

$$\frac{d^2y}{dx^2} = \frac{1}{t \cos^3 t}$$

$$R = \frac{(1 + \left(\frac{dy}{dx}\right)^2)^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left(1 + \left(\frac{\sin t}{\cos t}\right)^2\right)^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t \cos^3 t}}$$

$$R = \frac{\left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t \cos^3 t}}$$

$$R = \frac{\left(\frac{1}{\cos^2 t}\right)^{3/2}}{\frac{1}{t \cos^3 t}}$$

$$R = \frac{1/\cos^3 t}{1/t \cos^3 t}$$

$$R = \frac{1}{\cos^3 t} \times t \cos^3 t$$

$$\underline{\underline{R = t}}$$

3) expressions for the coordinates (h, k) of the Centre of Curvature in terms of t

Ans

$$h = x_1 - R \sin \theta$$

$$k = y_1 + R \cos \theta$$

$$R = b, \quad \theta = \tan^{-1} \left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{\sin b}{\cos b} = \tan b$$

Eqn

$$\theta = t$$

$$h = x, -t \sin t$$

$$k = y, t \cos t$$

$$\text{but } x = \cos t + t \sin t$$

$$y = \sin t - t \cos t$$

$$h = \cos t + t \sin t - t \sin t$$

$$h = \underline{\cos t}$$

$$k = \sin t - t \cos t + t \cos t$$

$$k = \sin t$$

$$(h, k) = (\underline{\cos t}, \underline{\sin t})$$