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 Course: ENGINEERING MATHS
 MATRIC: 1616NC05/034

$$d^2y/dx^2 - dy/dx - 2y = 8$$

$$y_h = d^2y/dx^2 - dy/dx - 2y = 0$$

$$y = e^{kx}$$

$$y' = ke^{kx}$$

$$y'' = k^2e^{kx}$$

$$k^2e^{kx} - ke^{kx} - 2e^{kx} = 0$$

$$(k^2 - k - 2)e^{-kx} = 0$$

$$y = e^{kx}$$

$$k^2 - k - 2 = 0$$

$$\cancel{k} + k - 2 = 0 \quad k^2 + k - 2k - 2 = 0$$

$$k(k+1) - 2(k+1) = 0$$

$$(k-2)(k+1) = 0$$

$$k_1 = 2 \quad k_2 = -1$$

$$y_h = C_1y_1 + C_2y_2$$

$$y_h = C_1e^{2x} + C_2e^{-x}$$

$$y_p = A$$

$$y = 0$$

$$y'' = 0$$

$$0 + 0 + 2A = 8$$

$$2A = 8 \quad \therefore A = 4$$

$$y = y_h + y_p$$

$$y = C_1e^{2x} + C_2e^{-x} + 4$$

$$d^2y/dx^2 - 4y = 10e^{3x}$$

$$y = e^{kx} \quad y' = ke^{kx} \quad y'' = k^2e^{kx}$$

$$k^2e^{kx} - 4e^{kx} = 10e^{3x}$$

$$(k^2 - 4)e^{kx} = 10e^{3x}$$

$$k^2 - 4 = 0 \quad k = \pm 2 \quad \therefore k_1 = 2, k_2 = -2$$

$$y = C_1 y_1 + C_2 y_2$$

$$y = C_1 e^{2x} + C_2 e^{-2x}$$

$$y = Ae^{2x}$$

$$y' = 2Ae^{2x}$$

$$y'' = 4Ae^{2x}$$

$$4Ae^{2x} + 3Ae^{2x} + 4Ae^{2x} = 10e^{2x}$$

$$11Ae^{2x} = 10e^{2x}$$

$$11A = 10$$

$$A = \frac{10}{11}$$

$$y = Ce^{2x} + Ge^{-2x} + 2e^{2x}$$

$$3) \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x}$$

$$(k^2 + 2k + 1)e^{2x} = 0$$

$$y = e^{2x}$$

$$k^2 + 2k + 1 = 0$$

$$k^2 + 2k + 1 = 0$$

$$k(k+1) + 1(k+1) = 0$$

$$k = -1 \text{ (twice)}$$

$$y = C_1 e^{-x} + C_2 x e^{-x}$$

$$y = C_1 e^{-x} + C_2 x e^{-x}$$

$$y_p = Ae^{-2x}$$

$$y_p' = -2Ae^{-2x}$$

$$y_p'' = 4Ae^{-2x}$$

$$4Ae^{-2x} - 2Ae^{-2x} + Ae^{-2x} = e^{-2x}$$

$$4Ae^{-2x} - 2Ae^{-2x} + Ae^{-2x} = e^{-2x}$$

$$Ae^{-2x} = e^{-2x}$$

$$A = 1$$

$$y = y_1 + y_p = (C_1 + C_2 x)e^{-x} + e^{-2x}$$

VISTALINE

$$4) \frac{d^2y}{dx^2} + 25y = 5x^2 + 1$$

$$y = e^{kx}$$

$$y' = k e^{kx}$$

$$y'' = k^2 e^{kx}$$

$$(k^2 + 25)e^{kx} = 0$$

$$y = e^{kx}$$

$$k^2 + 25 = 0$$

$$k^2 = -25$$

$$k = \pm 5i$$

$$k_1 = 5i, k_2 = -5i$$

$$y = C_1 e^{5ix} + C_2 e^{-5ix}$$

$$y = C_1 \cos 5x + C_2 \sin 5x$$

$$P-I: y = Ax^2 + Bx + D$$

$$y' = 2Ax + B$$

$$y'' = 2A$$

$$2A + 25(Ax^2 + Bx + D) = 5x^2 + 1$$

$$2A + 25Ax^2 + 25Bx + 25D = 5x^2 + 1$$

$$25Ax^2 + 25Bx + 20(25D + 2A) = 5x^2 + 1$$

$$25A = 5$$

$$A = \frac{1}{5}$$

$$25B = 0$$

$$B = 0$$

$$25D + 2A = 1$$

$$25D + \frac{2}{5} = 1$$

$$25D = \frac{3}{5}$$

$$D = \frac{3}{125}$$

$$y = y_1 + y_p$$

$$y = C_1 \cos 5x + C_2 \sin 5x + \frac{1}{5}x^2 + \frac{3}{125} = \frac{2}{125}$$

$$5) \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2\sin x$$

$$(k^2 - 2k + 1)e^{kx} = 0$$

$$k^2 - 2k + 1 = 0$$

$$k^2 - 2k + 1 = 0$$

$$k(k-1) - 1(k-1) = 0$$

$$k = 1 \text{ (twice)}$$

$y'' - 2y' + 2y = 2e^{2x}$
 $y_p = A e^{2x}$
 $y'' = 4A e^{2x}$
 $y' = 2A e^{2x}$
 $y = A e^{2x}$

$(4A - 4A + 2A) = 2$
 $2A = 2$
 $A = 1$
 $y_p = e^{2x}$

$A = 0$
 $A = 0$
 $2A = 2$
 $A = 1$

$y_p = e^{2x}$

$y = C_1 e^{2x} + C_2 e^{2x} + e^{2x}$

$y'' - 2y' + 2y = 2e^{2x}$
 $e^{2x}(y'' - 2y' + 2y) = 2e^{2x}$
 $(y'' - 2y' + 2y) = 2$

$y'' - 2y' + 2y = 2$

$-2y' + 2y = 2$

$-2y' + 2y = 2$

$-2y' + 2y = 2$

$-2y' + 2y = 2$

$y_1 = 2e^{2x}$

$y_2 = 2e^{-2x}$

$y = C_1 y_1 + C_2 y_2$

$y = C_1 e^{2x} + C_2 e^{-2x}$

$y = e^{2x}(C_1 \cos(x) + C_2 \sin(x))$

VISTALINE

$y'' = A e^{-2x}$

$y' = -2A e^{-2x}$

$y = A e^{-2x}$

$(11A + 11(-2A) + 5(A)) = 2e^{2x}$

$11A - 22A + 5A = 2$

$A = 2$

$y_p = 2e^{-2x}$

$y = C_1 \cos(x) + C_2 \sin(x) + 2e^{-2x}$

$y = C_1 \cos(x) + C_2 \sin(x) + 2e^{-2x}$

$\frac{dy}{dx} = e^{2x}(C_2 \cos(x) - C_1 \sin(x)) - 2e^{2x}(C_1 \cos(x) + C_2 \sin(x)) - 4e^{2x}$

$x = 0, y = 1, \frac{dy}{dx} = 0$

$y = e^{2(0)}(C_1 \cos(0) + C_2 \sin(0)) + 2e^{-2(0)}$

$y = 1(C_1 + 0) + 2e^{0}$

$y = C_1 + 2$

$1 = C_1 + 2$

$C_1 = -1$

$2 = e^{2(0)}(C_2 \cos(0) - C_1 \sin(0)) - 2e^{2(0)}(C_1 \cos(0) + C_2 \sin(0)) - 4e^{2(0)}$

$-2 = (C_2) - (C_1) - 4$

$2 = C_2 - C_1$ $\text{real } C_1 = -1$

$2 = C_2 - (-1)$

$2 = C_2 + 1$

$C_2 = 1$

$y = e^{-2x}(-1 \cos(x) + 1 \sin(x)) + 2e^{2x}$

VISTALINE

$$7 \quad 3 \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - y = 2x - 3$$

$$(3k^2 - 2k - 1)e^{kx} = 0$$

$$3k^2 - 2k - 1 = 0$$

$$3k^2 - 3k + k - 1 = 0$$

$$3k(k-1) + (k-1) = 0$$

$$(3k+1)(k-1) = 0$$

$$k_1 = -\frac{1}{3}, k_2 = 1$$

$$y_h = C_1 y_1 + C_2 y_2$$

$$y_h = C_1 e^{-x/3} + C_2 e^{x}$$

$$y_p = Ax + B$$

$$y' = A$$

$$y'' = 0$$

$$3(0) - 2(A) - Ax + B = 2x - 3$$

$$-2A - Ax + B = 2x - 3$$

$$-Ax + B - 2A = 2x - 3$$

$$-A = 2 \quad B - 2A = -3$$

$$A = -2 \quad B + 4 = -3$$

$$B = -7$$

$$y_p = -2x - 7$$

$$y = C_1 e^{-x/3} + C_2 e^x - 2x - 7$$

$$8 \quad \frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 8y = 8e^{4x}$$

$$y = e^{kx}$$

$$(k^2 - 6k + 8)e^{kx} = 8e^{4x}$$

$$k^2 - 6k + 8 = 0$$

$$k^2 - 2k - 4k + 8 = 0$$

$$k(k-2) - 4(k-2) = 0$$

$$(k-4)(k-2) = 0$$

$$k_1 = 4, k_2 = 2$$

$$y = C_1 y_1 + C_2 y_2$$

$$y = C_1 e^{4x} + C_2 e^{2x}$$

$$y_p = Ae^{4x}$$

$$y_p = 4Ae^{4x}$$

$$y_p = 16Ae^{4x}$$

$$(16A + 4A) = 8e^{4x}$$

$$21A = 8$$

$$A = \frac{8}{21}$$

$$y_p = \frac{8}{21} e^{4x}$$

$$y = C_1 e^{4x} + C_2 e^{2x} + \frac{8}{21} e^{4x}$$