

Patrick Eneke Obianwuzi

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Chemical Engineering

$$1) \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

$$m^2 - m - 2 = 0$$

$$m^2 - 2m + m - 2 = 0$$

$$(m^2 - 2m)(m - 2) = 0$$

$$m(m - 2) + 1(m - 2) = 0$$

$$(m + 1)(m - 2) = 0$$

$$m_1 = -1 \quad \& \quad m_2 = 2$$

$$C.F = y = Ae^{-x} + Be^{2x}$$

$$P.I: y = C$$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

$$0 - 0 - 2C = 8$$

$$-2C = 8$$

$$C = -4$$

$$P.I: y = -4$$

$$G.S: y = Ae^{-x} + Be^{2x} - 4$$

$$2) \frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \pm 2$$

$$C.F = y = Ae^{2x} + Be^{-2x}$$

$$P.I = y = Ce^{3x}$$

$$\frac{dy}{dx} = 3Ce^{3x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{3x}$$

$$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

$$9C - 4C = 10$$

$$5C = 10$$

$$C = 2$$

$$P.I = y = 2e^{3x}$$

$$G.S: Ae^{2x} + Be^{-2x} + 2e^{3x}$$

$$3) \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$(m+1)(m+1) = 0$$

$$m = -1$$

$$C.F: y = e^{-x}(A+Bx)$$

$$P.I = y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$4C - 4C + C = 1$$

$$C = 1$$

$$P.I = y = e^{-2x}$$

$$G.S = e^{-x}(A+Bx) + e^{-2x}$$

$$4) \frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$m^2 + 25 = 0$$

$$m = \pm 5i \quad m^2 = -25$$

$$m = i \pm 5 \quad \pm j 5$$

$$C.F = y = A \cos 5x + B \sin 5x$$

$$P.I = y = Cx^2 + Dx + E + Fx + G$$

$$\frac{dy}{dx} = 2Cx + D + F$$

$$\frac{d^2y}{dx^2} = 2C$$

$$2C + 25Cx^2 + 25Dx + 25E + 25Fx + 25G = 5x^2 + x$$

$$25Cx^2 + (25D + 25F)x + (2C + 25E + 25G) = 5x^2 + x$$

$$25C = 5$$

$$C = 5$$

$$25D + 25F = 1 \quad \text{--- (1)}$$

$$D + F = \frac{1}{25}$$

$$2C + 25E + 25G = 0$$

$$10 + 25E + 25G = 0$$

$$E + G = \frac{-10}{25} = \frac{-2}{5}$$

$$P.I = y = 5x^2 + \frac{1}{25}x + \frac{-2}{5}$$

$$G.S = A \cos 5x + B \sin 5x + 5x^2 + \frac{1}{25}x + \frac{-2}{5}$$

$$5) \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

$$m^2 - 2m + 1 = 0$$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - 1(m-1) = 0$$

$$(m-1)(m-1) = 0$$

$$m = 1$$

$$C.F = y = e^x(A + Bx)$$

$$P.I = y = C\cos x + D\sin x$$

$$\frac{dy}{dx} = -C\sin x + D\cos x$$

$$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$$

$$-C\cos x - D\sin x + 2\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$$

$$\cos x(-C - 2D + C) + \sin x(-D + 2C + D) = 4\sin x$$

$$\cos x(-2D) + \sin x(2C) = 4\sin x$$

$$-2D = 0$$

$$D = 0$$

$$2C = 4$$

$$C = 2$$

$$P.I : y = 2\cos x$$

$$G.S = y = e^{2x}(A + Bx) + 2\cos x$$

b) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that at $x=0$, $y=1$ and $\frac{dy}{dx} = -2$

$$m^2 + 4m + 5 = 0$$

$$a=1, b=4, c=5$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 4(1)(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm \sqrt{-1} \sqrt{4}}{2}$$

$$= \frac{-4 \pm \sqrt{1} \cdot 2}{2}$$

$$= -2 \pm j$$

$$\alpha = -2, \beta = 1$$

$$y = e^{-2x} (A \cos x + B \sin x)$$

P.I. $y = Ce^{-2x}$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} = 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$$

$$4C = 8C + 5C = 2$$

$$C = 2$$

P. I. $y = 2e^{-2x}$

$$G.S. \Rightarrow y = e^{-2x} (A \cos x + B \sin x) + 2e^{-2x}$$

at $x=0$, $y=1$ and $\frac{dy}{dx} = -2$

$$1 = e^{-2(0)} (A \cos 0 + B \sin 0) + 2e^{-2(0)}$$

$$1 = A + 2$$

$$A = -1$$

$$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) - 2e^{-2x}(A\cos x + B\sin x) - 4e^{-2x}$$

$$-2 = e^{-2(0)}(-A\sin 0 + B\cos 0) - 2e^{-2(0)}(A\cos 0 + B\sin 0) - 4e^{-2(0)}$$

$$-2 = 1(B) - A - 4$$

$$-2 = B - A - 4$$

$$2 = B - A$$

$$2 = B + 1$$

$$B = 1$$

$$y = e^{-2x}(-1\cos x + \sin x) + 2e^{-2x}$$

$$7) \cdot 3 \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - y = 2x - 3$$

$$3m^2 - 2m - 1 = 0$$

$$3m^2 + m - 3m - 1 = 0$$

$$m(3m+1) - 1(3m+1) = 0$$

$$(m-1)(3m+1) = 0$$

$$m_1 = 1 \text{ \& } m_2 = -\frac{1}{3}$$

$$\text{C.F. : } y = Ae^x + Be^{-\frac{x}{3}}$$

$$\text{P.I. : } y = Cx + D - E$$

$$\frac{dy}{dx} = C$$

$$\frac{d^2y}{dx^2} = 0$$

$$-2C - Cx + D - E = 2x - 3$$

$$-Cx - 1(C + D - E) = 2x - 3$$

$$2 = -C$$

$$C = -2$$

$$C + D - E = -3$$

$$-2 + D - E = -3$$

$$D - E = -1$$

$$P.I: y = -2x - 1$$

$$G.S: y = Ax^2 + Be^{-x/3} - 2x - 1$$

$$8) \frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 8y = 8e^{4x}$$

$$m^2 - 6m + 8 = 0$$

$$m^2 - 2m - 4m + 8 = 0$$

$$m(m-2) - 4(m-2) = 0$$

$$m(m-4)(m-2) = 0$$

$$m_1 = 4 \text{ \& } m_2 = 2$$

$$C.F: y = Ae^{4x} + Be^{2x}$$

$$P.I y = Ce^{4x}$$

$$\frac{dy}{dx} = 4Ce^{4x} \quad Cx(4e^{4x}) + Ce^{4x}$$

$$\frac{d^2y}{dx^2} = 16Ce^{4x} \quad 4Ce^{4x} \quad Cx(16e^{4x}) + C(4e^{4x}) + C4e^{4x}$$

$$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$$

$$16c - 24c + 8c = 8$$

$$Cx(16e^{4x}) + C(4e^{4x}) + C4e^{4x} - 6Cx(4e^{4x}) - 6Ce^{4x} + 8Ce^{4x} = 8e^{4x}$$

$$16cx + 4c + 4c - 24cx - 6c + 8cx = 8$$

$$16cx + 2c - 24cx + 8cx = 8$$

$$2c = 8$$

$$c = 4$$

$$P.I: y = \cancel{4c} + 4xe^{4x}$$

$$G.S: y = Ae^{4x} + Be^{2x} + 4xe^{4x}$$