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DEPT: CHEMICAL AND PETROLEUM ENGINEERING

MATRIC NO: 13/ENG01/009

COURSE CODE: CHE 531

COURSE TITLE: PROCESS DYNAMICS AND CONTROL 1

Assignment one

Answer

Let $\frac{d^2y}{dt^2} = \ddot{y}$ and $\frac{dy}{dt} = \dot{y}$

Therefore, we have

$$\ddot{y} - 3\dot{y} + 2y = 2e^{3t}$$

Taking laplace transform of both sides,

$$L[\ddot{y}] - 3L[\dot{y}] + 2L[y] = 2L[e^{3t}]$$

$$s^2y(s) - sy(0) - \dot{y}(0) - 3[sy(s) - y(0)] + 2y(s) = 2\left[\frac{1}{s-3}\right]$$

$$y(0) = 5, \qquad \dot{y}(0) = 7$$

$$s^2y(s) - 5s - 7 - 3sy(s) + 3(5) + 2y(s) = \frac{2}{s-3}$$

$$s^2y(s) - 3sy(s) + 2y(s) - 5s + 15 - 7 = \frac{2}{s-3}$$

$$s^2y(s) - 3sy(s) + 2y(s) - 5s + 8 = \frac{2}{s-3}$$

$$y(s)[s^2 - 3s + 2] - 5s + 8 = \frac{2}{s-3}$$

$$y(s)[s^2 - 3s + 2] = \frac{2}{s-3} - 8 + 5s$$

$$y(s)[s^2 - 3s + 2] = \frac{2 - 8(s-3) + 5s(s-3)}{s-3}$$

$$y(s)[s^2 - 3s + 2] = \frac{2 - 8s + 24 + 5s^2 - 15s}{s-3}$$

$$y(s)[s^2 - 3s + 2] = \frac{5s^2 - 23s + 26}{s-3}$$

$$y(s) = \frac{5s^2 - 23s + 26}{(s-3).(s^2 - 3s + 2)}$$

$$y(s) = \frac{(s-2).(s-\frac{13}{5})}{(s-3).(s^2 - 3s + 2)}$$

$$y(s) = \frac{\cancel{(s-2)}.(5s-13)}{(s-3).(s-1).\cancel{(s-2)}}$$

$$y(s) = \frac{5s - 13}{(s-3)(s-1)}$$

$$\frac{5s - 13}{(s-3)(s-1)} = \frac{A}{s-3} + \frac{B}{s-1}$$

$$\frac{5s - 13}{(s-3)(s-1)} = \frac{A(s-1) + B(s-3)}{(s-3)(s-1)}$$

$$5s - 13 = As - A + Bs - 3B$$

$$5s - 13 = (A+B)s - A - 3B$$

Comparing coefficient and using elimination method we have

$$A + B = 5 \qquad (1)$$

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$$A + 3B = 13 \qquad (2)$$

$$-2B = -8$$

$$B = \frac{-8}{-2} = 4$$

Substituting in equation (1)

$$A + B = 5$$

$$A = 5 - 4 = 1$$

$$y(s) = \frac{5s - 13}{(s-3)(s-1)} = \frac{1}{s-3} + \frac{4}{s-1}$$

$$y(t) = L^{-1}[y(s)] = L^{-1}\left[\frac{1}{s-3}\right] + 4L^{-1}\left[\frac{1}{s-1}\right]$$

$$y(t) = e^{3t} + 4e^{t}$$