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ENG381 Assignment 1

① $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

assume, $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m^2 + m - 2m - 2 = 0$

$m(m + 1) - 2(m+1) = 0$

$(m+1)(m-2) = 0$

$m = -1$ or $m = 2$

since the roots are real and distant,
the $C.F = Ae^{-x} + Be^{2x}$

since $f(x) = 8$, $y = C$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$\therefore 0 - 0 - 2(c) = 8$

$-2(c) = 8$

$c = -4$

$\therefore P.I = y = -4$

general solution =

$y = Ae^{-x} + Be^{2x} - 4$

② $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

(2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$$\frac{d^2y}{dx^2} - 4y = 0$$

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \pm 2$$

$CF = A\cosh 2x + B\sinh 2x$

$= A\cosh 2x + B\sinh 2x$

since $f(x) = 10e^{3x}$, $y = ce^{3x}$

$$\frac{dy}{dx} = 3ce^{3x}$$

$$\frac{d^2y}{dx^2} = 9ce^{3x}$$

$$9ce^{3x} - 4(ce^{3x}) = 10e^{3x}$$

$$9ce^{3x} - 4ce^{3x} = 10e^{3x}$$

$$5ce^{3x} = 10e^{3x}$$

$$5c = 10$$

$$c = 2$$

$\therefore P.I = y = 2e^{3x}$

general solution =

$y = A\cosh 2x + B\sinh 2x + 2e^{3x}$

(3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$(m+1)(m+1) = 0$$

$m = -1$ twice

$CF = e^{-x}(A + Bx)$

since $f(x) = e^{-2x}$, $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$C = 1$

$\therefore y = e^{-2x}$

general solution =

$y = e^{-x}(A + Bx) + e^{-2x}$

(4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm 5j$

~~CF = Acosh~~ $C.F = A\cos 5x + B\sin 5x$

$f(x) = 5x^2 + x$, $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 + 25Dx + 2C + 25E = 5x^2 + x$

by comparing coefficients

$25C = 5$
$C = 1/5$

$25D = 1$
$D = 1/25$

$2C + 25E = 0$
$2\left(\frac{1}{5}\right) + 25E = 0$

$25E = -\frac{2}{5}$

$E = \frac{-2}{25 \times 5}$

$E = \frac{-2}{125}$

PI =
$y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

general solution
$y = A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

(5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1) = 0$

$m = 1$ twice

$C.F = e^{x}(A + Bx)$

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since $f(x) = 4\sin x$

$$y = A\cos x + B\sin x$$

$$\frac{dy}{dx} = -A\sin x + B\cos x$$

$$\frac{d^2y}{dx^2} = -A\cos x - B\sin x$$

$$-A\cos x - B\sin x - 2(-A\sin x + B\cos x) + A\cos x + B\sin x = 4s$$

$$-A\cos x - B\sin x + 2A\sin x - 2B\cos x + A\cos x + B\sin x = 4\sin x$$

$$(\cancel{-A} - 2B \cancel{+A})\cos x + (2A \cancel{-B} \cancel{+B})\sin x = 4\sin x$$

$$-2B\cos x + 2A\sin x = 4\sin x$$

$$2A = 4 \quad , A = 2$$

$$-2B = 0$$

$$B = 0$$

$$\therefore P.I = 2\cos x + 0\sin x$$

$$= 2\cos x$$

general solution =

$$y = e^{x}(A + Bx) + 2\cos x$$

(6) $$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

given that at $x = 0$, $y = 1$, $dy/dx =$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$

$$m^2 + 4m + 5 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{(4)^2 - (4)(1)(5)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$\frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm 2j}{2}$$

$$= -2 \pm j$$

$C.F = e^{-2x}(A\cos x + B\sin x)$

since $f(x) = 2e^{-2x}$, $y = Ce^{-2x}$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$\cancel{2e}\ Ce^{-2x} = 2e^{-2x}$

$C = 2$

$P.I = 2e^{-2x}$

general solution,

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

when $x = 0$, $y = 1$

$(1) = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-2(0)}$

$1 = 1(A(1) + B(0)) + 2$

$1 = A + 2$

$A = -1$

when $x = 0$ $dy/dx = -2$

$$\frac{dy}{dx} = \frac{d(e^{-2x}(A\cos x + B\sin x))}{dx} + \frac{d}{dx}(2e^{-2x})$$

let $u = e^{-2x}$ $\qquad v = A\cos x + B\sin x$

$$\frac{dy}{dx} = -2e^{-2x} \qquad \frac{dv}{dx} = -A\sin x + B\cos x$$

$= (A\cos x + B\sin x) \cdot -2e^{-2x} + (-A\sin x + B\cos x)e^{-2x}$

$\frac{d}{dx}(2e^{-2x}) = -4e^{-2x}$

$\frac{dy}{dx} = (A\cos x + B\sin x) \cdot -2e^{-2x} + (-A\sin x + B\cos x)e^{-2x} - 4e^{-2x}$

when $x = 0$ $dy/dx = -2$

$-2 = (A\cos(0) + B\sin(0)) \times -2e^{-2(0)} + (-A\sin(0) + B\cos(0))e$
$4e^{2(0)}$

$-2 = (A + 0) \times -2(1) + (0 + B)1 - 4(1)$

$-2 = (-1 + 0)2 + (B) - 4$

$-2 = -2 + B - 4$

$-2 = -6 + B$

$B = 4$

$\therefore y = e^{-2x}(-\cos x + 4\sin x) + 2e^{-2x}$

(7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$$

$$\frac{d^2y}{dx^2} - \frac{2}{3}\frac{dy}{dx} - \frac{y}{3} = 0$$

$$m^2 - \frac{2}{3}m - \frac{1}{3} = 0$$

$$3m^2 - 2m - 1 = 0$$

$$3m^2 - 3m + m - 1 = 0$$

$$3m(m-1) + 1(m-1) = 0$$

$$(m-1)(3m+1) = 0$$

$$m_1 = 1 \quad . \quad m_2 = -\frac{1}{3}$$

since the roots are real and distant,

$C.F = y = Ae^{x} + Be^{-\frac{1}{3}x}$

since $f(x) = 2x - 3$, $y = Cx + D$.

$$\frac{dy}{dx} = C$$

$$\frac{d^2y}{dx^2} = 0$$

$$= 3(0) - 2(C) - Cx - D = 2x - 3$$

$$-2C - Cx - D = 2x - 3$$

$$-Cx - 2C - D = 2x - 3$$

comparing coefficients.

$$-2C - D = -3 \qquad -Cx = 2x$$

$$C = -2$$

$$= -2(-2) - D = -3$$

$$4 - D = -3$$

$$D = 7$$

$\therefore$ P.I = $-2x + 7$

general solution,

$y = Ae^{x} + Be^{-1/3 x} + (-2x + 7)$

$y = Ae^{x} + Be^{-1/3 x} - 2x + 7$

(3) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

assume $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 2m - 4m + 8 = 0$

$m(m-2) - 4(m-2) = 0$

$(m-2)(m-4) = 0$

$m_1 = 2, \quad m_2 = 4$

since the roots are real and distant

$C.F = Ae^{2x} + Be^{4x}$

since $f(x) = 8e^{4x}$, $y = Cxe^{4x}$

$\frac{dy}{dx} =$

$u = Cx \qquad v = e^{4x}$

$\frac{du}{dx} = C \qquad \frac{dv}{dx} = 4e^{4x}$

$\frac{dy}{dx} = Ce^{4x} + Cx4e^{4x}$

$\frac{d^2y}{dx^2} = 4Ce^{4x} + 16Cxe^{4x} + 4Ce^{4x}$

$4Ce^{4x} + 16Cxe^{4x} + 4Ce^{4x} - 6Ce^{4x} - 24Cxe^{4x} + 8Cxe^{4x} = 8e^{4x}$

$4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$

$8Ce^{4x} - 6Ce^{4x} = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$2C = 8$

$C = 4$

$\therefore P.I = 4xe^{4x}$

general solution,

$y = Ae^{2x} + Be^{4x} + 4xe^{4x}$.