1). $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2 = 0$

$m_1 = 2$ & $m_2 = -1$

C.F

$y = Ae^{2x} + Be^{-x}$

P.I $\Rightarrow y = C$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2C = 8$

$C = -4$

G.S

$y = Ae^{2x} + Be^{-x} - 4$

2). $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$m^2 - 4 = 0 \Rightarrow m = \pm\sqrt{4}$

$m = \pm 2$

C.F

$y = Ae^{2x} + Be^{-2x}$

P.I

$y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} + 0(3Ce^{3x}) - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

e^{3x} cancels out.

$9C - 4C = 10$

$C = 10/5$

$C = 2$

$y = 2e^{3x}$

G.S

$y = Ae^{2x} + Be^{-2x} + 2e^{3x}$

3). $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = e^{-2x}$

$m^2 + 2m + 1 = 0$

$x_1 = -1$ & $x_2 = 1$

C.F

$y = Ae^{-x} + Be^{x}$

P.I

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$C = 1$

$y = e^{-2x}$

G.S

$y = Ae^{-x} + Be^{x} + e^{-2x}$

5). $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$m = 1$ twice

C.F

$y = e^{x}(A\cos x + B\sin x)$

P.I

$y = C\cos x + B\sin x$

$\frac{dy}{dx} = -C\sin x + B\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - B\sin x$

$-C\cos x - B\sin x - 2(-C\sin x + B\cos x) + C\cos x + B\sin x = 4\sin x$

$-C\cos x - B\sin x + 2C\sin x - 2B\cos x + C\cos x + B\sin x = 4\sin x$

$2C\sin x - 2B\cos x = 4\sin x$

Comparing coefficients.

$2C = 4$

$C = 2$

$-2B = 0$

$B = 0$

$y = 2\cos x + 0\sin x$

$y = 2\cos x$

G.S

$\underline{y = e^x(A\cos x + B\sin x) + 2\cos x.}$

4. $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$m^2 + 25 = 0 \Rightarrow m = \pm\sqrt{25}$

$m = \pm 5$

C.F

$y = Ae^{5x} + Be^{-5x}$

P.I

$y = Cx^2 + Dx + E + fx + G$

$y = Cx^2 + x(D+f) + (E+G)$

$\frac{dy}{dx} = 2Cx + (D+F)$

$\frac{d^2y}{dx^2} = 2C$

Take $D+F = N$ & $E+G = P$

$2C + 25Cx^2 + 25Nx + 25P = 5x^2 + x$

Comparing coefficients.

$25C = 5$

$C = 1/5$

$25N = 1$

$N = 1/25$

$2C + 25P = 0$

$2(1/5) + 25P = 0$

$25P = -\frac{2}{5}$

$P = \frac{-2}{125}$

④ Contd.

$y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$y = 0.25x^2 + 0.04x - 0.016$

G.S

$\underline{y = Ae^{5x} + Be^{-5x} + 0.25x^2 + 0.04x - 0.016}$

6. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$m^2 + 4m + 5 = 0$

$m_1 = -2+i$ & $m_2 = -2-i$

C.F

$y = e^{-2x}(A\cos x + B\sin x)$

P.I

$y = Cxe^{-2x}$

$\frac{dy}{dx} = -2Cxe^{-2x} + Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Cxe^{-2x} - 2Ce^{-2x} - 2Ce^{-2x}$

$= 4Cxe^{-2x} - 4Ce^{-2x}$

$4Cxe^{-2x} - 4Ce^{-2x} + 4(-2Cxe^{-2x} + Ce^{-2x}) + 5(Cxe^{-2x}) = 2e^{-2x}$

$4Cxe^{-2x} - 4Ce^{-2x} - 8Cxe^{-2x} + 4Ce^{-2x} + 5Cxe^{-2x} = 2e^{-2x}$

$9Cx - 8Cx = 2$

$Cx = 2$

$y = 2e^{-2x}$

G.S

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

At $x=0$ & $y=1$

$1 = e^{-2(0)}[A\cos(0) + B\sin(0)] + 2e^{-2(0)}$

$1 = A + 2 \Rightarrow A = -1$

At $x=0$ & $\frac{dy}{dx} = 2$

$\frac{dy}{dx} = -2e^{-2x}(A\cos x + B\sin x) + e^{-2x}(-A\sin x + B\cos x)$

$-4e^{-2x}$

6). Contd.

$2 = -2e^{-2(0)}[A\cos(0) + B\sin(0)] + e^{-2(0)}[-A\sin(0) + B\cos(0)]$

$-4e^{-2(0)}$

$2 = -2A + B - 4$ Recall $A = -1$

$2 = 2 + B - 4$

$B = 4$

Therefore G.S will be

$y = e^{-2x}(-\cos x + 4\sin x) + 2e^{-2x}$

7). $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$m^2 - 2m - 1 = 0$

$m_1 = 1+\sqrt{2}$, & $m_2 = 1-\sqrt{2}$

C.F

$y = Ae^{(1+\sqrt{2})x} + Be^{(1-\sqrt{2})x}$

P.I

$y = Cx + D + E$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$-2C - Cx - (D+E) = 2x - 3$

Let $D+E = K$

$-2C - Cx - K = 2x - 3$

Comparing coefficients

$C = -2$

$-2C - K = -3$

$-2(-2) - K = -3$

$-K = -7$

$K = 7$

$y = -2x + 7 \Rightarrow y = 7 - 2x$

G.S

$y = Ae^{(1+\sqrt{2})x} + Be^{(1-\sqrt{2})x} + 7 - 2x$

8). $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$m_1 = 4$ & $m_2 = 2$

C.F

$y = Ae^{4x} + Be^{2x}$

P.I

$y = Cxe^{4x}$

$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 8Ce^{4x}$

$16Cxe^{4x} + 8Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8Cxe^{4x} = 8e^{4x}$

$16Cx + 8C - 24Cx - 6C + 8Cx = 8$

$24Cx - 24Cx + 8C - 6C = 8$

$2C = 8$

$C = 4$

$y = 4xe^{4x}$

G.S

$y = Ae^{4x} + Be^{2x} + 4xe^{4x}$