16/ENG05/020

2. Determine whether each of the following series is convergent.

$$\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$$

$$U_n = \frac{x}{(n+1)(n+2)}$$

$$U_{n+1} = \frac{2}{(n+2)(n+3)}$$

$$\lim_{n\to\infty} \left| \frac{U_{n+1}}{U_n} \right| = \left| \frac{x}{(n+1)(n+2)} \times \frac{(n+2)(n+1)}{x} \right|$$

$$\lim_{n\to\infty} = \left| \frac{1 + 1/n}{1 \quad 3/n} \right| = \frac{1}{1} = n$$

$\therefore$ since $\lim_{n\to\infty} \left| \frac{U_{n+1}}{U_n} \right| = 1$

$\therefore U_n$ is inconclusive or determined convergent

Using P-series to compare with the series, P-series $\frac{1}{n^p}$

$$U_n = \frac{x}{(n+1)(n+2)} \quad \text{for } P = 2.$$

$$\frac{x}{(n+1)(n+2)} > \frac{1}{n^2}$$

Test for $n = 1, 2, 3, 4$

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