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MATRIC NO:- 16/ENG03/062
DEPARTMENT :- CIVIL ENGINEERING
COURSE CODE: CVE ENG 381

ASSIGNMENT I

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Solution

$m^2 - m - 2 = 0$

Using the quadratic formula

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2}$

$= -1$ or 2

$m_1 = 2$ $m_2 = -1$

$y = Ae^{2x} + Be^{-x}$

P.I:

$y = c$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2c = 8$

$c = -4$

General Solution:

$y = Ae^{2x} + Be^{-x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Solution

$m^2 - 4 = 0$

$(m+2)(m-2) = 0$

$m_1 = -2$ $m_2 = +2$

CF $y = Ae^{2x} + Be^{-2x}$

PI

$y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$9C - 4C = 10$

$C = 2$

$\therefore y = 2e^{3x}$

GS

$y = Ae^{2x} + Be^{-2x} + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$m^2 + 2m + 1 = 0$

Using quadratic formula

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{4-4}}{2} = \frac{-2 \pm 0}{2} = -1$ twice.

$y = e^{-x}(A + Bx)$

PI

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x}, e^{-2x}$

$4C - 4C + C = 1$

$C = 1$

$y = e^{-2x}$

GS

$y = e^{-x}(A + Bx) + e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$m^2 + 25 = 0$

$(m-5)(m+5) = 0$

$m = 5$ and $m = -5$

$y = Ae^{5x} + Be^{-5x}$

PI

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + Dx + E = 5x^2 + x$

$25Cx^2 = 5x^2$, $Dx = x$, $2C + E = 0$

$C = 5$, $D = 1$, $E = -10$

$\therefore y = 5x^2 + x - 10$

GS

$y = Ae^{5x} + Be^{-5} + 5x^2 + x - 10$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

Using quadratic formula

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{2 \pm \sqrt{2^2-4}}{2} = \frac{2 \pm 0}{2} = 1$ twice

$y = e^x(A + Bx)$

PI

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$\sin x(-D + 2C + D) + \cos x(C - 2D - C) = 4\sin x$

$-D + 2C + D = 4$, $C - 2D - C = 0$

$C = 2$, $D = 0$

$y = 2\cos x$

GS

$y = e^x(A + Bx) + 2\cos x$

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6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$, given that at $x=0$, $y=1$ and $\frac{dy}{dx} = -2$

Solution

$m^2 + 4m + 5 = 0$

Using quadratic equation

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm 2j}{2}$

$m = -2+j$ or $-2-j$

$y = e^{-2x}(A\cos x + B\sin x)$

PI

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 5C = 2$

$C = 2$

$\therefore y = 2e^{-2x}$

GS

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

when $x=0$, $y=1$

$1 = A + 2 \Rightarrow A = -1$

when $x=0$, $\frac{dy}{dx} = -2$

~~$\frac{dy}{dx} = -2e^{-2x}(-A\sin x + B\cos x$~~ $\frac{dy}{dx} = -2e^{-2x}A\cos x + A\sin x e^{-2x} + -2e^{-2x}B\sin x + B\cos x e^{-2x}$

$+ -4e^{-2x}$

$-2 = -2A + B - 4$

$-2 = 2 + B$ where $A = -1$

$B = 0$

GS $\Rightarrow y = e^{-2x}(-\cos x) + 2e^{-2x}$

7) $3\frac{d^2y}{dx^2} - \frac{dy}{dx} - y = 2x - 3$

$m^2 - 2m - \frac{1}{3} = 0$

Using quadratic equation

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{2 \pm \sqrt{4 + \frac{4}{3}}}{6} = \frac{2 \pm 4}{6} = 1 \text{ or } -\frac{1}{3}$

$y = Ae^x + Be^{x/3}$

PI

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx} = 2C$

$3(2C) - 2(2Cx + D) - (Cx^2 + Dx + E) = 2x - 3$

$Cx^2 - 4Cx - Dx + 6C - 2D - E = 2x - 3$

$C = 0, \quad -4C - D = 2, \quad 6C - 2D - E = -3$

$\therefore D = -2 \qquad E = 7$

PI $y = -2x + 7$

GS

$y = Ae^x + Be^{x/3} - 2x + 7$

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$m_1 = 4 \text{ and } m_2 = 2$

$y = Ae^{2x} + Be^{4x}$

PI

$y = Ce^{4x} \qquad \frac{dy}{dx} = 4Ce^{4x} \qquad \frac{d^2y}{dx^2} = 16Ce^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

$16C - 24C + 8C = 8$

$C = 0$

GS

$y = Ae^{2x} + Be^{4x}$