

Electrostatic

$$1.) \frac{d^2y}{dx^2} - 2y = 8$$

$$m^2 - m - 2 = 0$$

C.F.; $m^2 - m - 2 = 0$

$$m(m-2) + 1(m-2) = 0$$

$$m = -1 \text{ or } 2$$

$$y = Ae^{-x} + Be^{2x}$$

P.I.; let $y = c$

$$\frac{dy}{dx} = 0; \frac{d^2y}{dx^2} = 0$$

$$0 - 0 - 2c = 8$$

$$-2c = 8; c = -4 = y$$

C.S.; $y = Ae^{-x} + Be^{2x} - 4$

$$2.) \frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

C.F.; $m^2 - 4 = 0$

$$m = \pm 2$$

$$y = C \cosh 2x + D \sinh 2x$$

P.I.; let $y = Ce^{3x}$

$$\frac{dy}{dx} = 3Ce^{3x}; \frac{d^2y}{dx^2} = 9Ce^{3x}$$

$$\text{So, } 9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$$

$$5Ce^{3x} = 10e^{3x}$$

$$C = 2; y = 2e^{3x}$$

C.S.; $y = C \cosh 2x + D \sinh 2x + 2e^{3x}$

$$3.) \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

C.F.; $m^2 + 2m + 1 = 0$

$$m = -1; y = e^{-x}(A + Bx)$$

P.I.; let $y = Ce^{-2x}; \frac{dy}{dx} = -2Ce^{-2x}; \frac{d^2y}{dx^2} = 4Ce^{-2x}$

$$\text{So, } 4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$Ce^{-2x} = e^{-2x}$$

$$C = 1; y = e^{-2x}$$

C.S.; $y = e^{-x}(A + Bx) + e^{-2x}$

$$4.) \frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

C.F.; $m^2 = -25; m = \pm 5j$

$$y = C \cos 5x + D \sin 5x$$

P.I.; let $y = Cx^2 + Dx + E$

$$\frac{dy}{dx} = 2Cx + D; \frac{d^2y}{dx^2} = 2C$$

$$\text{So, } 2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$25x^2 + 25Dx + 2L + E = 5x^2 + Dx$$

$$25L = 5; L = \frac{1}{5}$$

$$25D = 1; D = \frac{1}{25}$$

$$2L + E = 0$$

$$\frac{2}{5} + E = 0; E = -\frac{2}{5}$$

$$25x + 25E = 0; E = -25x \times \frac{1}{25}$$

$$E = -\frac{2}{125}$$

$$y = \frac{2x^2}{5} + \frac{x}{25} - \frac{2}{125} = \frac{1}{125} (25x^2 + 5x - 2)$$

$$\text{P.S.; } y = C \cos 5x + D \sin 5x + \frac{1}{125} (25x^2 + 5x - 2)$$

So;

$$(-C \cos x - D \sin x) - 2(-C \sin x + D \cos x) + C \cos x +$$

$$D \sin x = 4 \sin x$$

$$-C \cos x + C \cos x - D \sin x + D \sin x + 2C \sin x = 4 \sin x$$

$$-2D \cos x = 4 \sin x$$

$$2L = 4; L = 2$$

$$-2D = 0; D = 0$$

$$y = 2 \cos x + 0$$

P.S.;

$$y = e^{2x} (A + Bx) + 2 \cos x$$

$$5.) \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = 4 \sin x$$

$$\text{C.F.; } m^2 - 2m + 1 = 0$$

$$m = 1 = m_1 = m_2$$

$$y = e^x (A + Bx)$$

$$\text{P.I.; } y = C \cos x + D \sin x$$

$$\frac{dy}{dx} = -C \sin x + D \cos x$$

dx

$$\frac{d^2y}{dx^2} = -C \cos x - D \sin x$$

$$6.) \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 5y = 2e^{-2x}; \text{ also } y = 1$$

$$\frac{dy}{dx} = -2$$

$$\text{C.F.; } m^2 + 4m + 5 = 0$$

$$m = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$$

$$m = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$$

$$y = e^{-2x} (A \cos x + B \sin x)$$

P.I., let $y = ce^{-2x}$

$$\frac{dy}{dx} = -2ce^{-2x} ; \frac{d^2y}{dx^2} = 4ce^{-2x}$$

so, $4ce^{-2x} + 4(-2ce^{-2x}) + 5ce^{-2x} = 2e^{-2x}$

$$4ce^{-2x} - 8ce^{-2x} + 5ce^{-2x} = 2e^{-2x}$$

$$ce^{-2x} = 2e^{-2x}$$

$$c = 2 ; y = 2e^{-2x}$$

C.S.I., $y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

if $y = 1$ when $x = 0$

$$1 = A + 2$$

$$A = -1$$

$$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) - 2e^{-2x}(A\cos x + B\sin x) - 4e^{-2x} = 2$$

$$-2 = e^{-2(0)}(-A\sin 0 + B\cos 0) - 2e^{-2(0)}(A\cos 0 + B\sin 0) - 4e^{-2(0)}$$

$$-2 = B - 2A - 4$$

$$2 = B - 2A \quad A = -1$$

$$2 = B + 2$$

$$B = 0$$

$$y = e^{-2x}(-\cos x) + 2e^{-2x}$$

$$= e^{-2x}(-\cos x + 2) = e^{-2x}(2 - \cos x)$$

7.) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

C.F.

$$3m^2 - 7m - 1 = 0$$

$$3m^2 - 3m + m - 1 = 0$$

$$3m(m-1) + 1(m-1) = 0$$

$$m = 1 \text{ or } -\frac{1}{3}$$

$$y = Ae^{x} + Be^{-\frac{x}{3}}$$

P.F., let $y = (x + D)$

$$\frac{dy}{dx} = C ; \frac{d^2y}{dx^2} = 0$$

$$\text{so, } 3(0) - 2C - (x + D) = 2x - 3$$

$$-2C - (x + D) = 2x - 3$$

$$-2C - 2x - D = 2x - 3$$

$$C = -2$$

$$-(2C + D) = -3$$

$$-4 + D = 3$$

$$D = 7$$

$$y = -2x + 7$$

$$\text{G.S. } y = Ae^{x} + Be^{-\frac{x}{3}} - 2x + 7$$

$$\text{6. } \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

C.F.

$$m^2 - 6m + 8 = 0$$

$$(m-4)(m-2)$$

$$m = 4 \text{ or } 2$$

$$y = Ae^{4x} + Be^{2x}$$

P.F., let $y = Ce^{4x}$

$$\frac{dy}{dx} = 4Ce^{4x} ; \frac{d^2y}{dx^2} = 16Ce^{4x}$$

$$\text{so, } 16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$$

$$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$$

$$C = 0$$

$$\text{G.S. } y = \underline{\underline{Ae^{4x} + Be^{2x}}}$$