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12/ENG02/043

COMPUTER ENGINEERING

ENG 381

$$1. \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

$$m^2 - m - 2 = 0$$

$$m^2 + m - 2m - 2 = 0$$

$$m(m+1) - 2(m+1) = 0$$

$$m - 2 = 0$$

$$m + 1 = 0$$

$$\therefore m_1 = 2$$

$$m_2 = -1$$

C.F

$$y = Ae^{2x} + Be^{-x}$$

$$P.I = y = c$$

$$\therefore \frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

$$0 - 0 - 2c = 8$$

$$-2c = 8$$

$$c = -4$$

General Solution

$$G.S = C.F + P.I$$

$$y = Ae^{2x} + Be^{-x} - 4$$

$$②. \frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

$$\frac{d^2y}{dx^2} - 4y = 0$$

$$m^2 - 4 = 0$$

$$m = \pm 2$$

$$y = A \cosh 2x + B \sinh 2x$$

$$P.I. = y = Ce^{3x}$$

$$\frac{dy}{dx} = 3Ce^{3x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{3x}$$

$$9Ce^{3x} + 0(3Ce^{3x}) - 4(Ce^{3x}) = 10e^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

$$5Ce^{3x} = 10e^{3x}$$

$$C = \frac{10e^{3x}}{5e^{3x}}$$

$$C = 2$$

$$1. \quad y = 2e^{3x}$$

General equation (G.E) = C.F + P.I.

$$y = A \cosh 2x + B \sinh 2x + 2e^{3x}$$

$$(3) \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$m = -1 \text{ (twice)}$$

$$\therefore \text{C.F. } y = e^{-x}(A+Bx)$$

$$\therefore \text{P.I. } y = Ce^{-3x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} + 2[-2Ce^{-2x}] + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$C = 1$$

$$y = 1e^{-2x}$$

$$y = e^{-2x}$$

General solution (G.S) = C.F + P.I

$$y = e^{-x}(A+Bx) + e^{-2x}$$

A. $\frac{d^2y}{dx^2} + 25y = 5x^2 + 2$

$$\frac{d^2y}{dx^2} + 25y = 0$$

$$m^2 + 25 = 0$$

$$m^2 + 0 + 25 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-0 \pm \sqrt{0^2 - 4(1)(25)}}{2(1)}$$

$$\frac{-0 \pm \sqrt{-25 \times 4}}{2}$$

$$\frac{-0 \pm \sqrt{-1} \sqrt{4} \sqrt{25}}{2}$$

$$m = \frac{\pm j 2\sqrt{25}}{2}$$

$$m = \pm j\sqrt{25} \quad \therefore m = \pm j5$$

C.F.

$$y = A \cos 5x + B \sin 5x$$

$$P.I \Rightarrow y = Cx^2 + Dx + E$$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$$2C + 0(Cx + D) + 25(Cx^2 + Dx + E) = 5x^2 + 2$$

$$2C + 25(Cx^2 + Dx + E) = 5x^2 + 2$$

$$2C + 25Cx^2 + 75Dx + 25E = 5x^2 + 2$$

comparing coefficients

$$2C + 25E = 0 \quad \text{--- (i)}$$

$$25Cx^2 = 5$$

$$C = \frac{5}{25}$$

$$C = \frac{1}{5} \quad \text{--- (ii)}$$

$$250 = 1$$

$$0 = \frac{1}{25} \text{ --- (1)}$$

substituting eqn (1) into (1)

$$2\left(\frac{1}{5}\right) + 25E = 0$$

$$\frac{2}{5} + 25E = 0$$

$$\frac{1}{25} \times 25E = -\frac{2}{5} \times \frac{1}{25}$$

$$E = -\frac{2}{125}$$

$$\therefore P.I = \frac{5x^2}{25} + \frac{1}{25}x - \frac{2}{125}$$

$$-\frac{1}{125} \left[25x^2 + 5x - 2 \right]$$

General solution

$$y = A \cos 5x + B \sin 5x + \frac{1}{125} \left[25x^2 + 5x - 2 \right]$$

$$5. \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4 \sin x$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$$m^2 - 2m + 1 = 0$$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - m(m-1) = 0$$

$$-(m-1) = 0 \text{ [twice]}$$

$$m = 1 \text{ twice}$$

$$y = e^x [A + Bx]$$

$$P.I. = C \cos x + D \sin x = y$$

$$\frac{dy}{dx} = -C \sin x + D \cos x$$

$$\frac{d^2y}{dx^2} = -C \cos x - D \sin x$$

$$[-C \cos x - D \sin x] - 2[-\sin x + D \cos x] + [C \cos x + D \sin x] = 4 \sin x$$

$$-C \cos x - D \sin x + 2 \sin x - 2D \cos x + C \cos x + D \sin x = 4 \sin x$$

$$-C \cos x - 2D \cos x + C \cos x - D \sin x + 2 \sin x + D \sin x = 4 \sin x$$

$$\cos x [-C - 2D + C] + \sin x [-D + 2 + D] = 4 \sin x$$

$$\cos x [-2D] + \sin x [2] = 4 \sin x$$

Compare coefficients

$$-2D = 0$$

$$D = 0$$

$$2C = 4$$

$$C = 2$$

$$P.T = 2 \cos x + 0(\sin x)$$

$$y = 2 \cos x$$

$$\therefore G.S = y = e^x [A + Bx] + 2 \cos x$$

$$6. \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 5y = 2e^{-2x}$$

given that (i) $x=0, y=1$ (ii) $\frac{dy}{dx} = -2$

$$\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 5y = 0$$

$$m^2 + 4m + 5 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

$$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$m = \frac{-4 \pm \sqrt{-4}}{2}$$

$$m = \frac{-4}{2} + \frac{j2}{2}$$

$$m = -2 \pm j$$

$$y = e^{-2x} (A \cos x + B \sin x)$$

$$\therefore P.I. ; y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{dy}{dx} = 4e^{-2x}$$

$$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$$

$$4Ce^{-2x} + 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$$

$$Ce^{-2x}(4+8+5) = 2e^{-2x}$$

$$Ce^{-2x} = 2e^{-2x}$$

$$\therefore C = 2$$

$$P.I. = y = 2e^{-2x}$$

$$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-x}$$

$$\therefore \text{at } x=0 \quad y=1$$

$$1 = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-0}$$

$$1 = 1(A+0) + 2$$

$$1 = A + 2$$

$$A = -2 + 1$$

$$A = -1$$

$$\therefore \text{at } x=0, \quad \frac{dy}{dx} = -2$$

$$\frac{dy}{dx} = -2e^{-2x}(A\sin x + B\cos x) - 2e^{-x}$$

$$-2 = -2e^{-2(0)}(-A\sin(0) + B\cos(0)) - 2e^{-0}$$

$$-2 = -2(0+B) - 2$$

$$-2 = -2B - 2$$

$$-2B = -2 + 2$$

$$B = 0/-2$$

$$B = 0$$

$$y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-x}$$

$$y = -e^{-2x}\cos x + 2e^{-x}$$

$$7. \quad 3 \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} - y = 2x - 3$$

$$3 \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} - y = 0$$

$$3m^2 - 2m - 1 = 0$$

$$3m^2 - 3m + m - 1 = 0$$

$$3m(m-1) + 1(m-1) = 0$$

$$(3m+1)(m-1) = 0$$

$$3m+1 = 0$$

$$3m = -1$$

$$m_1 = -\frac{1}{3}$$

$$m-1 = 0$$

$$m_2 = 1$$

$$\text{C.F. } y = Ae^{-\frac{1}{3}x} + Be^x$$

$$\text{P.I. } y = Cx + D$$

$$\frac{dy}{dx} = C$$

$$\frac{d^2 y}{dx^2} = 0$$

$$3(0) + -2(C) - [Cx + D] = 2x - 3$$

$$0 - 2C - Cx - D = 2x - 3$$

$$-2C - D - Cx = 2x - 3$$

Comparing coefficients

$$-2C - D = -3$$

$$-C = 2$$

$$\therefore C = -2$$

$$-2(-2) - D = -3$$

$$4 - D = -3$$

$$-D = -3 - 4$$

$$-D = -7$$

$$D = 7$$

$$\therefore \text{P.I. } y = -2x + 7$$

\(\therefore\) G.S.

$$y = Ae^{-\frac{1}{3}x} + Be^x - 2x + 7$$

$$8. \frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 8y = 8e^{4x}$$

$$\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 8y = 0$$

$$m^2 - 6m + 8 = 0$$

$$m^2 - 2m - 4m + 8 = 0$$

$$m(m-2) + 4(m-2) = 0$$

$$m-4 = 0$$

$$m-2 = 0$$

$$m_1 = 4$$

$$m_2 = 2$$

$$y = Ae^{4x} + Be^{2x}$$

$$\text{P.I. ; } y = Ce^{4x}$$

$$\frac{dy}{dx} = 4Ce^{4x}$$

$$\frac{d^2 y}{dx^2} = 16Ce^{4x}$$

$$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$$

$$Ce^{4x}(16 - 24 + 8) = 8e^{4x}$$

$$Ce^{4x}(0) = 8e^{4x}$$

$$C = \frac{8e^{4x}}{e^{4x}(0)}$$

$$y = 0e^{4x}$$

$$y = 0$$

G.G ;

$$y = Ae^{4x} + Be^{2x} //$$