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1. $$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

$$m^2 - m - 2 = 0$$

$$m(m+1) - 2(m+1) = 0$$

$$m = 2, -1$$

$$y = Ae^{-x} + Be^{2x}$$

$$y = c$$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

$$0 - 0 - 2c = 8$$

$$-2c = 8$$

$c = -4$ (particular integral)

$\therefore Ae^{-x} + Be^{2x} - 4$ (General Solution)

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(8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m(m-2) - 4(m-2) = 0$

$m = 4, 2$

$y = Ae^{2x} + Be^{4x}$

$y = Cxe^{4x}$

$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$16Cxe^{4x} - 24Cxe^{4x} + 8Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$2C = 8$

$C = 4$

$y = 4xe^{4x}$ (Particular integral)

$\therefore$ General Solution $= Ae^{2x} + Be^{4x} + 4xe^{4x}$

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⑦ $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$m = -\frac{1}{3}, 1$

$Ae^{x} + Be^{-\frac{1}{3}x}$

$y = Cx + D$ (Particular integral)

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$-2C - Cx - D = 2x - 3$

$-C = 2$

$C = -2$

$-2C - D = -3$

$2C + D = 3$

$2(-2) + D = 3$

$-4 + D = 3$

$D = 7$

$\therefore y = Ae^{x} + Be^{-\frac{1}{3}x} - 2x + 7$

$y = 2e^{-2x}$ (particular integral)

$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$

at $x = 0$ and $y = 1$

$1 = e^{-2(0)}[C\cos(0) + D\sin(0)] + 2e^{-2(0)}$

$1 = 1[C + 0] + 2$

$1 = C + 2$

$C = -1$

$\frac{dy}{dx} = [e^{-2x}(-C\sin x + D\cos x)] + [2e^{-2x}(C\cos x + D\sin x)] - 4e^{-2x}$

when $\frac{dy}{dx} = -2$, $x = 0$

$-2 = [D] + [-2C] - 4$

$-2 = D - 2C - 4$

$D - 2C = 2$

$D = 2 + 2[-1]$

$D = 0$

$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$

$y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-2x}$

$y = e^{-2x}(-\cos x + 2)$

$\therefore$ Particular Solution $\Rightarrow y = \underline{e^{-2x}(2 - \cos x)}$

(6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$, $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$, $a = 1$, $b = 4$, $c = 5$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm j2}{2}$

$m = -2 \pm j$

$y = e^{-2x}(C\cos x + D\sin x)$ (particular integral)

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4[-2Ce^{-2x}] + 5[Ce^{-2x}] = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 5C = 2$

$C = 2$

(5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$m^2 - 2m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)$ twice

$m = \frac{1}{2}$

$y = e^x(A + Bx)$ (particular integral)

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$-C\cos x - D\sin x - 2[-C\sin x + D\cos x] + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$-C\cos x - 2D\cos x + C\cos x - D\sin x + 2C\sin x + D\sin x = 4\sin x$

$\cos x(-C - 2D + C) + \sin x(-D + 2C + D) = 4\sin x$

$-C - 2D + C = 0$

$-2D = 0$

$D = 0$

$-D + 2C + D = 4$

$C = 2$

$y = 2\cos x + D\sin x = 2\cos x$ (particular integral)

General Solution $= \underline{e^x(A + Bx) + 2\cos x}$

(4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm j5$

$y = C\cos 5x + D\sin 5x$

$y = Cx^2 + Dx + E$, $\frac{dy}{dx} = 2Cx + D$, $\frac{d^2y}{dx^2} = 2C$

$2C + 25[Cx^2 + Dx + E] = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25C = 5$

$C = 1/5$

$25D = 1$

$D = 1/25$

$2C + 25E = 0$

$2[1/5] + 25E = 0$

$2/5 + 25E = 0$

$25E = -2/5$

$E = -2/125$

$y = 1/5\, x^2 + 1/25\, x - 2/125$ (particular integral)

$\therefore$ General Solution $= C\cos 5x + D\sin 5x + 1/5\, x^2 + 1/25\, x - 2/125$

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③ $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

$m^2 + 2m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)$ twice

$y = e^{-x}(A + Bx)$

$y = ce^{-2x}$

$\frac{dy}{dx} = -2ce^{-2x}$

$\frac{d^2y}{dx^2} = 4ce^{-2x}$

$4ce^{-2x} + 2(-2ce^{-2x}) + ce^{-2x} = e^{-2x}$

$4ce^{-2x} - 4ce^{-2x} + ce^{-2x} = e^{-2x}$

$ce^{-2x} = e^{-2x}$

$c = 1$

$c = e^{-2x}$ (particular integral)

General Solution $= e^{-x}(A + Bx) + e^{-2x}$

② $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm 2$

$y = C\cosh 2x + D\sinh 2x$

$y = ce^{3x}$

$\frac{dy}{dx} = 3ce^{3x}$

$\frac{d^2y}{dx^2} = 9ce^{3x}$

$9ce^{3x} - 4ce^{3x} = 10e^{3x}$

$5ce^{3x} = 10e^{3x}$

Divide through by e^{3x}

$5c = 10$

$c = 2$

$c = 2e^{3x}$ (particular integral)

$\therefore C\cosh 2x + D\sinh 2x + 2e^{3x}$ (General Solution)