

Question 1

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

C.F

$$m^2 - m - 2 = 0$$

$$(m-2)(m+1) = 0$$

$$m = 2 \quad m = -1$$

$$m_1 = 2 \quad m_2 = -1$$

$$y = Ae^{m_1x} + Be^{m_2x}$$

$$y = Ae^{2x} + Be^{-x}$$

P.I

$$y = C$$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow 0 - 0 - 2C = 8$$

$$2C = -8$$

$$C = \frac{-8}{2}$$

$$C = -4 \quad \therefore y = -4 \quad \text{--- P.I}$$

$$\text{G.S} = \text{C.F} + \text{P.I}$$

$$= Ae^{2x} + Be^{-x} + (-4)$$

$$= Ae^{2x} + Be^{-x} - 4$$

Question 2

$$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

C.F

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \sqrt{4}$$

$$m = \pm 2$$

$$y = A \cosh 2x + B \sinh 2x$$

P.I

$$y = Ce^{3x}$$

$$\frac{dy}{dx} = 3Ce^{3x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{3x}$$

$$\Rightarrow 9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

$$9C - 4C = 10$$

$$5C = 10$$

$$C = 10/5 = 2 \therefore y = 2e^{3x} \quad \text{--- P.I}$$

$$\therefore GF = CF + PI$$

$$= A \cosh 2x + B \sinh 2x + 2e^{3x}$$

Question 3

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

C.F

$$m^2 + 2m + 1 = 0$$

$$(m+1)(m+1) = 0$$

$$m = -1 \text{ twice}$$

$$y = e^{-x} (A + Bx)$$

P.I

$$y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

dx

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

dx²

$$4ce^{-2x} + 2(-2ce^{-2x}) + ce^{-2x} = e^{-2x}$$

$$4ce^{-2x} - 4ce^{-2x} + ce^{-2x} = e^{-2x}$$

$$ce^{-2x} = e^{-2x}$$

$$c = 1 \therefore y = e^{-2x} - P \cdot I$$

$$\therefore G.S = C.F + P \cdot I$$

$$= e^{-2x}(A+Bx) + e^{-2x}$$

Question 4

$$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

C.F

$$m^2 + 25 = 0$$

$$m^2 = -25$$

$$m = \sqrt{-25}$$

$$= \sqrt{-1} \sqrt{25}$$

$$m = j5$$

$$y = A \cos nx + B \sin nx$$

$$= A \cos 5x + B \sin 5x$$

P.I

$$y = Cx^2 + Dx + E$$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$$\Rightarrow 2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$$

Comparing coefficients

$$25C = 5$$

$$C = \frac{1}{5}$$

$$25D = 1$$

$$D = \frac{1}{25}$$

$$2C + 25E = 0$$

$$2\left(\frac{1}{5}\right) + 25E = 0$$

$$25E = -\frac{2}{5}$$

$$E = -\frac{2}{125}$$

$$\Rightarrow \text{P.I.} : y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$
$$= \frac{25x^2 + 5x - 2}{125}$$

$$G.S = C.F + P.I$$

$$= A \cos 5x + B \sin 5x + \frac{25x^2 + 5x - 2}{125}$$

Question 5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

C.F

$$m^2 - 2m + 1 = 0$$

$$(m-1)(m-1) = 0$$

$m = 1$ twice

$$y = e^x (A + Bx)$$

P.I

$$y = C \cos x + D \sin x$$

$$\frac{dy}{dx} = -C \sin x + D \cos x$$

$$\frac{d^2y}{dx^2} = -C \cos x - D \sin x$$

$$\Rightarrow -C \cos x - D \sin x - 2(-C \sin x + D \cos x) + C \cos x + D \sin x = 4 \sin x$$

$$-C \cos x - D \sin x + 2C \sin x - 2D \cos x + C \cos x + D \sin x = 4 \sin x$$

$$2C \sin x - 2D \cos x = 4 \sin x$$

Comparing coefficients

$$2C = 4$$

$$C = \frac{4}{2} \Rightarrow C = 2$$

$$2D = 0$$

$$D = 0$$

$$\text{P.I } y = 2 \cos x + 0 \sin x = 2 \cos x$$

$$G.S = e^x (A + Bx) + 2 \cos x$$

Question 6

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}, \text{ given that at } x=0, y=1$$

C.F

$$m^2 + 4m + 5 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm \sqrt{-1} \sqrt{4}}{2}$$

$$= \frac{-4 \pm j2}{2}$$

$$= -2 \pm j$$

$$m_1 = -2 + j \quad m_2 = -2 - j$$

$$y = e^{-2x} (A \cos x + B \sin x)$$

P.I

$$y = ce^{-2x}$$

$$\frac{dy}{dx} = -2ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4ce^{-2x}$$

$$4ce^{-2x} + 4(-2ce^{-2x}) + 5(ce^{-2x}) = 2e^{-2x}$$

$$4c - 8c + 5c = 2$$

$$c = 2$$

$$y = 2e^{-2x}$$

$$G.S = C.F + P.I$$

$$y = e^{-2x} (A \cos x + B \sin x) + 2e^{-2x}$$

$$\text{at } x=0, y=1$$

$$1 = e^{-2(0)} (A \cos 0 + B \sin 0) + 2e^{-2(0)}$$

$$1 = 1(A) + 2$$

$$1 = A + 2$$

$$A = 1 - 2$$

$$A = -1$$

$$\frac{dy}{dx} = e^{-2x} [-A \sin x + B \cos x] + [A \cos x + B \sin x] \cdot -2e^{-2x} + 2e^{-2x}$$

$$\frac{dy}{dx} = e^{-2x} (-A \sin x + B \cos x) - 2e^{-2x} (A \cos x + B \sin x) + 4e^{-2x}$$

$$\text{at } x = 0 \quad \frac{dy}{dx} = -2$$

$$-2 = e^{-2(0)} (-A \sin 0 + B \cos 0) - 2e^{-2(0)} (A \cos 0 + B \sin 0) + 4e^{-2(0)}$$

$$-2 = B - 2A - 4$$

$$-2 = B - 2(-1) - 4$$

$$B = -2 - 2 + 4$$

$$B = 0$$

$$\begin{aligned} p.s &= e^{-2x} (-\cos x) + 2e^{-2x} \\ &= -e^{-2x} \cos x + 2e^{-2x} \\ &= e^{-2x} (2 - \cos x) \end{aligned}$$

Question 7

$$3 \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - y = 2x - 3$$

C.F

$$3m^2 - 2m - 1 = 0$$

$$(3m+1)(m-1) = 0$$

$$3m = -1 \quad \text{or } m = 1$$

$$m_1 = -\frac{1}{3} \quad m_2 = 1$$

$$y = Ae^{-\frac{1}{3}x} + Be^x$$

P.F

$$y = Cx + D$$

$$\frac{dy}{dx} = C$$

dx

$$\frac{d^2y}{dx^2} = 0$$

$$3(0) - 2(C) - [Cx + D] = 2x - 3$$

$$0 - 2C - Cx - D = 2x - 3$$

Comparing coefficients

$$-C = 2$$

$$C = -2$$

$$-2C - D = -3$$

$$-2(-2) - D = -3$$

$$4 - D = -3$$

$$D = 7$$

$$\Rightarrow y = -2x + 7$$

: G.S

$$= Ae^{-\frac{1}{3}x} + Be^x - 2x + 7$$

Question 8

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

C.F

$$m^2 - 6m + 8 = 0$$

$$(m-4)(m-2) = 0$$

$$m_1 = 4 \quad m_2 = 2$$

$$y = Ae^{4x} + Be^{2x}$$

P.I

$$y = Cxe^{4x}$$

$$\frac{dy}{dx} = [x \cdot 4e^{4x} + e^{4x}]C$$

$$\frac{d^2y}{dx^2} = 4C[x \cdot 4e^{4x} + e^{4x}] + 4Ce^{4x}$$

$$= 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$$

$$\Rightarrow 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8Cxe^{4x} = 8e^{4x}$$

$$16Cx + 4C + 4C - 24Cx - 6C + 8Cx = 8$$

$$16Cx + 4C - 24Cx - 6C + 8Cx = 8$$

$$-8Cx + 2C + 8e^x = 8$$

$$2C = 8$$

$$C = \frac{8}{2}$$

$$C = 4$$

$$\therefore y = 4xe^{4x}$$

$$G.S = C.F + P.I$$

$$= Ae^{4x} + Be^{2x} + 4xe^{4x}$$