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14/ENG01/019
CHEMICAL ENGINEERING
CHE 531

Assignment I

Given that $y(0) = 5$ and $\dot{y}(0) = 7$

Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 2e^{3t}$

Soln

$$\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = s^2y(s) - sy(0) - \dot{y}(0)$$

$$\mathcal{L}\left[\frac{dy}{dt}\right] = sy(s) - y(0)$$

$$\mathcal{L}[y] = y(s)$$

$$\mathcal{L}[e^{3t}] = \frac{1}{s-3}$$

$$\mathcal{L}\left[\frac{d^2y}{dt^2}\right] - 3\mathcal{L}\left[\frac{dy}{dt}\right] + 2\mathcal{L}[y] = 2\mathcal{L}[e^{3t}]$$

$$s^2y(s) - sy(0) - \dot{y}(0) - 3[sy(s) - y(0)] + 2y(s) = \frac{2}{s-3} \quad \text{—— (1)}$$

Given that $y(0) = 5$ and $\dot{y}(0) = 7$, substitute into equ (1)

$$s^2y(s) - 5s - 7 - 3sy(s) + 15 + 2y(s) = \frac{2}{s-3}$$

Collecting like terms

$$s^2y(s) - 3sy(s) + 2y(s) - 5s + 8 = \frac{2}{s-3}$$

$$y(s)\left[s^2 - 3s + 2\right] = \frac{2}{s-3} + 5s - 8$$

$$y(s) = \frac{2}{s-3} + \frac{5s}{1} - \frac{8}{1} \cdot \frac{1}{s^2-3s+2}$$

$$y(s) = \frac{2 + 5s(s-3) - 8(s-3)}{s-3} \cdot \frac{1}{s^2-3s+2}$$

$$y(s) = \frac{2 + 5s^2 - 15 - 8s + 24}{s-3} \cdot \frac{1}{s^2-3s+2}$$

$$y(s) = \frac{5s^2 - 23s + 26}{s-3} \times \frac{1}{s^2 - 3s + 2}$$

Finding the roots of $5s^2 - 23s + 26$ and $s^2 - 3s + 2$, we have

$$y(s) = \frac{(5s-13)\cancel{(s-2)}}{(s-3)} \times \frac{1}{\cancel{(s-2)}(s-1)}$$

$$y(s) = \frac{(5s-13)}{(s-3)(s-1)}$$

Using Partial fraction

$$y(s) = \frac{5s-13}{(s-3)(s-1)} = \frac{A}{s-3} + \frac{B}{s-1}$$

$$= \frac{A(s-1) + B(s-3)}{(s-3)(s-1)}$$

$$y(s) = \frac{5s-13}{(s-3)(s-1)} = \frac{As - A + Bs - 3B}{(s-3)(s-1)}$$

$$5s - 13 = As + Bs - A - 3B$$

Equating coefficients and constants

$5 = A + B$ —— (1)

$13 = A + 3B$ —— (2)

$A + B = 5$ —— (3)

$A + 3B = 13$ —— (4)

Subtract equ (3) from (4)

$2B = 8$

$B = 4$

Substitute $B = 4$ into equ (3)

$\cancel{A+}$ $A + 4 = 5$

$A = 1$

$$y(s) = \frac{1}{s-3} + \frac{4}{s-1}$$

We then find laplace inverse

$$y(t) = \mathcal{L}^{-1}[y(s)] = \mathcal{L}^{-1}\left[\frac{1}{s-3}\right] + 4\mathcal{L}^{-1}\left[\frac{1}{s-1}\right]$$

$y(t) = e^{3t} + 4e^{t}$

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