ENG281

NWANKWO NWACHUKWU 16/ENG04/036 Elect/Elect 200level

$y = \sin\theta - \theta\cos\theta$

$x = \cos\theta + \theta\sin\theta$

$\frac{dy}{d\theta} = \cos\theta - (\theta\sin\theta + \cos\theta)$

$\frac{dy}{d\theta} = \cos\theta + \theta\sin\theta - \cos\theta$

$\frac{dy}{d\theta} = +\theta\sin\theta$

$\frac{dx}{d\theta} = -\sin\theta + (\theta\cos\theta + \sin\theta)$

$\frac{dx}{d\theta} = -\sin\theta + \theta\cos\theta + \sin\theta$

$= \theta\cos\theta$

$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{\theta\sin\theta}{\theta\cos\theta} = \tan\theta$

$\frac{d^2y}{dx^2} = \frac{d(\tan\theta)}{dx} \times \frac{d\theta}{dx} = \sec^2\theta \times \frac{1}{\theta\cos\theta} = \frac{1}{\cos^2\theta} \times \frac{1}{\cos\theta} \times \frac{1}{\theta}$

$= \frac{1}{\cos^3\theta} \times \frac{1}{\theta} = \frac{\sec^3\theta}{\theta}$

$\therefore \frac{d^2y}{dx^2} = \theta^{-1}\sec^3\theta$

$R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}} = \frac{(1 + (\tan\theta)^2)^{3/2}}{\theta^{-1}\sec^3\theta}$

$R = \frac{(1 + \tan^2\theta)^{3/2}}{\theta^{-1}\sec^3\theta} = \frac{(\sec^2\theta)^{3/2}}{\theta^{-1}(\sec\theta)^3} = \frac{(\sec\theta)^{2\times\frac{3}{2}}}{\theta^{-1}(\sec\theta)^3}$

$R = \frac{(\sec\theta)^3}{\theta^{-1}(\sec\theta)^3} = \frac{1}{\theta^{-1}} = \theta$ units

2) Centre of Curvature

$x_1 = a + R\sin\theta$, $b = y_1 + R\cos\theta$, $\theta = \tan^{-1}\left\{\frac{dy}{dx}\right\}$

$R = t$

Solve

$\theta = \tan^{-1}\left\{\frac{dy}{dx}\right\}$

$\theta = \tan^{-1}\{\tan t\}$

$\theta = t$

$x_1 = \cos t + t\sin t$

$\therefore \cos t + t\sin t = a + (t)\sin(t)$

$a = \cos t + t\sin t - t\sin t$

$a = \cos t$

$b = y_1 + R\cos\theta$

$y_1 = \sin t - t\cos t$

$\therefore b = \sin t - t\cos t + t\cos t$

$b = \sin t$

Centre of Curvature $= (a, b) = (\cos t, \sin t)$