

ENG281

NWANKWO NWACHUKWU 16/ENG104/036 Elect/Elect 200level

$$y = \sin t - t \cos t$$

$$x = \cos t + t \sin t$$

$$\frac{dy}{dt} = \cos t - (t \sin t + \cos t)$$

$$= \cos t + t \sin t - \cos t$$

$$\frac{dy}{dt} = \underline{\underline{t \sin t}}$$

$$\frac{dx}{dt} = -\sin t + (t \cos t + \sin t)$$

$$= -\sin t + t \cos t + \sin t$$

$$= t \cos t$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{t \sin t}{t \cos t} = \underline{\underline{\tan t}}$$

$$\frac{d^2y}{dx^2} = \frac{d(\tan t)}{dx} \times \frac{dt}{dx} = \sec^2 t \times \frac{1}{t \cos t} = \frac{1}{\cos^2 t} \times \frac{1}{\cos t} \times \frac{1}{t}$$

$$= \frac{1}{\cos^3 t} \times \frac{1}{t} = \underline{\underline{\frac{\sec^3 t}{t}}}$$

$$\therefore \frac{d^2y}{dx^2} = \underline{\underline{t^{-1} \sec^3 t}}$$

$$R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left(1 + (\tan t)^2\right)^{3/2}}{t^{-1} \sec^3 t}$$

$$R = \frac{\left(1 + \tan^2 t\right)^{3/2}}{t^{-1} \sec^3 t} = \frac{(\sec^2 t)^{3/2}}{t^{-1} (\sec t)^3} = \frac{(\sec t)^{2 \times \frac{3}{2}}}{t^{-1} (\sec t)^3}$$

$$R = \frac{(\sec t)^3}{t^{-1} (\sec t)^3} = \frac{1}{t^{-1}} = \underline{\underline{\frac{1}{t} \text{ units}}}$$

2) Centre of Curvature

$$x_1 = a + R \sin \theta, \quad y_1 = y_1 + R \cos \theta, \quad \theta = \tan^{-1} \left\{ \frac{dy}{dx} \right\}$$

$R = b$

Solve

$$\theta = \tan^{-1} \left\{ \frac{dy}{dx} \right\}$$

$$\theta = \tan^{-1} \{ \tan b \}$$

$$\theta = \underline{\underline{b}}$$

$$x_1 = \cos b + b \sin b$$

$$\therefore R \cos \theta + b \sin \theta = a + (b) \sin (b)$$

$$a = \cos b + \cancel{b \sin b} - \cancel{b \sin b}$$

$$a = \underline{\underline{\cos b}}$$

$$y_1 = y_1 + R \cos \theta$$

$$y_1 = \sin b - \cancel{b \cos b}$$

$$\therefore y_1 = \sin b - \cancel{b \cos b} + \cancel{b \cos b}$$

$$y_1 = \underline{\underline{\sin b}}$$

$$\text{Centre of Curvature} = \frac{(a, b)}{\underline{\underline{(\cos b, \sin b)}}}$$