Ass 1

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1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1 - 4(1 \times -2)}}{2(1)}$

$= \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2}$

$m_1 = \frac{-1+3}{2} = 1 \qquad m_2 = \frac{-1-3}{2} = -2$

$y = Ae^{x} + Be^{-2x} \Leftarrow$ C.F

P.I

$y = C$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$\therefore -2C = 8$

$C = -4$

$\Rightarrow y = -4$

Hence General Solution = C.F + P.I = $y = Ae^{x} + Be^{-2x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4$

$m = \pm 2$

$y = A\cosh 2x + B\sinh 2x \Leftarrow$ Complementary function

Partial integration

$y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$\therefore \quad 9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$C = 2$

$\therefore \quad y = 2e^{3x}$

General Solution;

$y = A\cosh 2x + B\sinh 2x + 2e^{3x}$ //

3) $\dfrac{d^2y}{dx^2} + \dfrac{2dy}{dx} + y = e^{-2x}$

Complementary function

$$\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = 0$$

$$m^2 + 2m + 1 = 0$$

$$\cancel{m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$\cancel{= \frac{-1 \pm \sqrt{4 - 4}}{2(1)}} \quad \cancel{= \frac{-1 \pm j\sqrt{3}}{2}} \quad \cancel{= \frac{-1}{2} + j\frac{\sqrt{3}}{2}}$$

$$(m+1)(m+1)$$

$$m = -1$$

$$y = e^{-x}(A + Bx)$$

Partial integration

$$y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = +4Ce^{-2x}$$

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$Ce^{-2x} = e^{-2x}$$

$$C = 1$$

$$\therefore y = e^{-2x}$$

General Solution;

$$y = e^{-x}(A + Bx) + e^{-2x}$$

4) $\dfrac{d^2y}{dx^2} + 25y = 5x^2 + x$

$\dfrac{d^2y}{dx^2} + 25y = 0$

Complementary function

$m^2 + 25 = 0$

$m^2 = 25$

$m = \pm 5$

$y = A\cos 5x + B\sin 5x$

Partial integration

$y = Cx^2 + Dx + E$

$\dfrac{dy}{dx} = 2Cx + D$

$\dfrac{d^2y}{dx^2} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

Comparing LHS & RHS

$25C = 5$ $\qquad$ $25D = 1$ $\qquad$ $2C + 25E = 0$

$C = 1/5$ $\qquad$ $D = 1/25$ $\qquad$ $\dfrac{2(1)}{5} + 25E = 0$

$E = \dfrac{-2}{5} \div 25 = \dfrac{-2}{125}$

$y = \dfrac{x^2}{5} + \dfrac{x}{25} - \dfrac{2}{125}$

General solution:

$y = A\cos 5x + B\sin 5x + \dfrac{x^2}{5} + \dfrac{x}{25} - \dfrac{2}{125}$ //

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

Complementary function

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$$m^2 - 2m + 1 = 0$$
$$(m-1)^2$$
$$m = 1$$

$$y = e^x (A + Bx)$$

Patial integration

$$y = C\cos x + D\sin x$$
$$\frac{dy}{dx} = -C\sin x + D\cos x$$
$$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$$

$$\Rightarrow -C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$$

$$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$$

$$\cos x(-C - 2D + C) + \sin x(-D + 2C + D) = 4\sin x$$

$$\cos x(-2D) + \sin x(2C) = 4\sin x$$

Comparing LHS -> RHS

$2C = 4$

$\cancel{D} \; -2D = 0$

$C = 2$

$D = 0$

$\therefore \; y = 2\cos x$

General solution;

$$y = e^{x}(A + Bx) + 2\cos x$$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

Complementary function

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$

$$m^2 + 4m + 5 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm j\sqrt{4}}{2} = -2 \pm j$$

$$y = e^{-2x}(A\cos x + B\sin x)$$

Partial integration

$$y = Ce^{-2x}$$

$$\frac{dy}{dx} = \cancel{-2e^{-2x}} \quad -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = \cancel{4e^{-2x}} \quad 4Ce^{-2x}$$

$$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$$

$$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$$

$$Ce^{-2x} = 2e^{-2x}$$

$$C = 2$$

$\therefore \quad y = 2e^{-2x}$

General Solution;

$$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$$

when $x = 0$, $y = 1$

$$\Rightarrow 1 = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-2(0)}$$

$$1 = 1(A) + 2$$

$$A = 1 - 2 = -1$$

$$\frac{dy}{dx} = \frac{d}{dx}(y)$$

$u = e^{-2x}$ $\qquad$ $V = A\cos x + B\sin x$

$\frac{du}{dx} = -2e^{-2x}$ $\qquad$ $\frac{dv}{dx} = -A\sin x + B\cos x$

$$e^{-2x}(-A\sin x + B\cos x) + (A\cos x + B\sin x)(-2e^{-2x})$$

$$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) + (A\cos x + B\sin x)\cdot(-2e^{-2x}) +$$
$$(-4e^{-2x})$$

when $\frac{dy}{dx} = -2$, $x = 0$, $y = 1$

$$\Rightarrow -2 = e^{-2(0)}(-A\sin(0) + B\cos(0)) + (A\cos 0 + B\sin 0)(-2e^{-2(0)}) - 4e^{-2(0)}$$

$-2 = 1(B + A(-2)) - 4$

$-2 = B - 2A - 4$

$2 = B - 2A$

$B = 2 + 2A$ $\qquad$ sub $A = -1$

$B = 2 + 2(-1)$

$B = 0$

$$y = e^{-2x}(\cos x + 0) = e^{-2x}(\cos x)$$

7) $\frac{3d^2y}{dx^2} - \frac{2dy}{dx} - y = 2x - 3$

$\frac{3d^2y}{} - \frac{2dy}{dx} - y = 0$

Complementary function

$3m^2 - 2m - 1 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3 \cdot -1)}}{2(3)}$

$= \frac{2 \pm \sqrt{4+12}}{6} = \frac{2 \pm \sqrt{16}}{6} = \frac{1}{3} \pm \frac{4}{6} = \frac{1}{3} \pm \frac{2}{3}$

$m_1 = \frac{1+2}{3} = 1 \qquad m_2 = \frac{1-2}{3} = \frac{-1}{3}$

$y = Ae^{x} + Be^{-x/3}$

Partial integration

$y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$3(0) - 2(C) - Cx - D = 2x - 3$

$-2C - Cx - D = 2x - 3$

Comparing LHS & RHS

$-C = 2$ $\qquad$ $-2C - D = -3$

$C = -2$ $\qquad$ $-2(-2) - D = -3$

$4 - D = -3$

$D = 4 + 3 = 7$

$\Rightarrow y = -2x + 7$

General Solution:

$$y = Ae^{x} + Be^{-\frac{x}{3}} - 2x + 7$$

8) $\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 8y = \cancel{4}\, 8e^{4x}$

$\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 8y = 0$

Complementary function

$m^2 - 6m + 8 = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1 \cdot 8)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 - 32}}{2} = \frac{6 \pm \sqrt{4}}{2} = 3 \pm 1$$

$m_1 = 3 + 1 = 4$ $\qquad$ $m_2 = 3 - 1 = 2$

$\Rightarrow \quad y = Ae^{2x} + Be^{4x}$

Partial integration

$$y = Ce^{4x}$$

$$\frac{dy}{dx} = 4Ce^{4x}$$

$$\frac{d^2y}{dx^2} = 16Ce^{4x}$$

$\Rightarrow \quad 16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$$

$$0C = 8e^{4x}$$

$$C = 0$$

$$y = 0$$

General Solution

$$y = Ae^{2x} + Be^{4x} \;//$$