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Mechanical Engineering
16/ENG 06/041
ENG 281

a. Evaluate $\lim_{x \to \pi/2} \left[\frac{x^2 - \pi/4 \sin(\cos x)}{x - \pi/2} \right]$

$= \frac{(\pi/2)^2 - \pi/4 \sin(\cos(\pi/4))}{\pi/2 - \pi/2} = \frac{0}{0}$ undefined

$\lim_{x \to \pi/2} \left\{ \frac{(2x - 0)(\sin)(-\sin x) + \cos x(\cos x)}{1 - 0} \right\}$

$= \frac{2(\pi/2)[-\sin^2(\pi/2) + \cos^2(\pi/2)]}{1}$

$= \pi(-1 + 0)$

$= \pi(-1)$

$\lim_{x \to \pi/2} \left[\frac{x^2 - \pi/4 \sin(\cos x)}{x - \pi/2} \right] = -\pi$

b. $\lim_{x \to \pi/2} \ln \left[\frac{\exp(3x^2 + 2x - 1)}{x + 1} \right]$

$\lim_{x \to \pi/2} \left[\frac{(3x - 1)(x + 1)}{(x + 1)} \right]$

$\lim_{x \to \pi/2} (3x - 1)$

$= 3(\pi/2) - 1$

$= 3\pi/2 - 1$

$\lim_{x \to \pi/2} \ln \left[\frac{\exp(3x^2 + 2x - 1)}{x + 1} \right] = \frac{3\pi - 2}{2}$

c. $\lim_{x \to 2 + \sqrt{3}} \cos \left[\sin^{-1} \frac{(x - 2)}{(x - \sqrt{3})} \right]$

$\cos \left[\sin^{-1} \frac{(2 + \sqrt{3} - 2)}{(2 + \sqrt{3} - \sqrt{3})} \right]$

$\cos \left[\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) \right]$

$\lim_{x \to 2 + \sqrt{3}} \cos \left[\sin^{-1} \frac{(x - 2)}{(x - \sqrt{3})} \right] = \cos 60 = 1/2$

d. $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

$= \frac{4^2 - 8(4) + 16}{4^2 - 5(4) + 4} = \frac{0}{0}$ undefined

$\lim_{x \to 4} \frac{2x - 8}{2x - 5}$

$= \frac{2(4) - 8}{2(4) - 5} = \frac{0}{3} = 0$

$\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right] = 0$

2a. $\frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \frac{2}{5 \times 6} + \dots$

$U_n = \frac{2}{(n+1) \times (n+2)}$, $U_{n+1} = \frac{2}{(n+1+1)(n+2+1)} = \frac{2}{(n+2)(n+3)}$

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$

$= \frac{n^2 + 3n + 2}{n^2 + 5n + 6}$

divide through by the highest power of n

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{\frac{n^2}{n^2} + \frac{3n}{n^2} + \frac{2}{n^2}}{\frac{n^2}{n^2} + \frac{5n}{n^2} + \frac{6}{n^2}}$

$= \frac{1 + 0 + 0}{1 + 0 + 0} = \frac{1}{1} = 1$

It maybe either convergent or divergent

using test 1 on U_n

$U_n = \frac{2}{n^2 + 3n + 2}$

$= \frac{\frac{2}{n^2}}{\frac{n^2}{n^2} + \frac{3n}{n^2} + \frac{2}{n^2}} = \frac{0}{1 + 0 + 0}$

$= 0$

converge

b. $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \dots$

$$U_n = \frac{2}{n^2}, \quad U_{n+1} = \frac{2}{(n+1)^2}$$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+1)^2} \times \frac{n^2}{2}$$

$$= \frac{n^2}{n^2+2n+1}$$

Divide by the highest power of n

$$= \frac{n^2/n^2}{n^2/n^2 + 2n/n^2 + 1/n^2} = \frac{1}{1+0+0} = \frac{1}{1}$$ may converge / diverge

Using test 1 on U_n

$$U_n = \frac{2}{n^2}$$

$$\frac{2/n^2}{n^2/n^2} = \frac{0}{1} = 0$$

Converge

c. $U_n = \frac{1+2n^2}{1+n^2}$

$$\lim_{n\to\infty} U_n = \frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2}$$

$$= \frac{0+2}{0+1} = \frac{2}{1}$$

$= 2$ diverge

3. $\frac{x}{27} + \frac{x^2}{125} + \dots + \frac{x^n}{(2n+1)^3}$

$$U_n = \frac{x^n}{(2n+1)^3}, \quad U_{n+1} = \frac{x^{n+1}}{(2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{(x)(2n+1)^3}{(2n+3)^3}$$

$$= \frac{(8n^3 + 12n^2 + 6n + 1)\, x}{8n^3 + 18n^2 + 54n + 27}$$

$$= \frac{8xn^3 + 12xn^2 + 6xn + x}{8n^3 + 18n^2 + 54n + 27}$$

divide by highest power of n.

$$= \frac{\frac{8xn^3}{n^3} + \frac{12xn^2}{n^3} + \frac{6xn}{n^3} + \frac{x}{n^3}}{\frac{8n^3}{n^3} + \frac{18n^2}{n^3} + \frac{54n}{n^3} + \frac{27}{n^3}}$$

$$= \frac{8x}{8}$$

$$\frac{8x}{8} < 1$$

$$8x < 8$$

$$x < 1$$

4. $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$

$\frac{\sin 0 - \cos 0}{0^3} = \frac{0 - 1}{0}$ undefined

$\lim_{x \to 0} \frac{\cos 0 + \sin 0}{3(0)^2} = \frac{1 + 0}{0}$ undefined

$\lim_{x \to 0} \frac{-\sin x + \cos x}{6x} = \frac{-\sin 0 + \cos 0}{6(0)} = \frac{-0 + 1}{0}$ undefined

$\lim_{x \to 0} \frac{-\cos x - \sin x}{6} = \frac{-\cos 0 - \sin 0}{6} = \frac{-1 - 0}{6} = -\frac{1}{6}$