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Chemical Engineering

Eng 281

1a) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$

$= \left[\left(\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \right) \sin(\cos \pi/2) \right]$

$= \frac{\left(\frac{\pi^2}{4} = \pi/4 \right) \sin\left(\cos \pi/2\right)}{0}$

thus underteminant. ∴ with L'hopital, $\delta y/\delta x$ of numerator $= u\, dv/dx + v\, du/dx$.

$\frac{dy}{dx} = \begin{cases} \text{let } u = x^2 - \pi/4 \\ v = \sin(\cos x) \end{cases}$

$dy/dx = 2x \qquad \frac{dv}{dx} = ?$

$\frac{d}{dx} \sin(\cos x) = \quad$ let $\cos x = w$

$V = \sin w$

$dv/dw = \cos w$

$$\frac{dw}{dx} = -\sin x.$$

$$\frac{dy}{dx} = \frac{dy}{dw} \times \frac{dw}{dx} = -\sin x \cos(\cos x)$$

$$= \frac{\left(x^2 - \frac{\pi}{4}\right) \times -\sin \cos(\cos x) + \sin(\cos x)(2x)}{1}$$

$$= \left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \times -\sin 90 \cos(\cos 90) + \sin \cos 90 \times 2\left(\frac{\pi}{2}\right)$$

$$= \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right) \times -1 + 0 \times \pi$$

$$= \frac{-\pi^2}{4} + \frac{\pi}{4} \; ; \; = \frac{\pi}{4} - \frac{\pi^2}{4}$$

$\therefore$ ~~$\lim_{x \to \pi/2} \left((x^2 - \pi/4) \right)$~~

$$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$$

$$= \frac{\pi(1-\pi)}{4}$$

Q1B

$$\lim_{x \to \pi/2} \ln \left[\exp \left(\frac{3x^2 + 2x - 1}{x+1} \right) \right]$$

$$= \lim \ln \left(\exp \left[\frac{(3x-1)(x+1)}{x+1^2} \right] \right)$$

$$= \lim_{x \to \pi/2} \ln \left[\exp (3x-1) \right]$$

$$= \lim_{x \to \pi/2} (3x-1) = 3(\pi/2) - 1$$

$$= \frac{3\pi}{2} - 1 = 3\pi/_2 - 1/_1$$

$$\lim \ln \left[\exp \frac{(3x^2 + 2x - 1)}{x+1} \right] = \frac{3\pi - 2}{2}$$

Q1C

$$\lim_{x \to 2+\sqrt{3}} \cos \left(\frac{\sin^{-1} (x-2)}{x - \sqrt{3}} \right)$$

$$\lim_{x \to 2+2\sqrt{3}} \cos \left[\sin^{-1} \left(\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3})} \right) \right]$$

$$\cos \left(\sin^{-1} \left[\frac{\sqrt{3}}{2} \right] \right)$$

$$= \cos \left[\sin^{-1} (0.8660) \right]$$

$$\Rightarrow \cos 60^\circ$$

$$= 1/2$$

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Q 1D

$$\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4}\right]$$

$$\lim_{x \to 4} \left[\frac{(x-4)(x-4)}{(x-4)(x-1)}\right]$$

$$\lim_{x \to} \left[\frac{x-4}{x-1}\right]$$

$$= \frac{4-4}{4-1} = 0/3 = 0$$

Q 2A

$$u_n = \frac{2}{(n+1)(n+2)}$$

$$u_n + 1 = \frac{2}{(n+2)(n+3)}$$

ratio $\frac{u_n+1}{u_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$

$$\frac{u_n+1}{u_n} = \frac{n+1}{n+3}$$

$$\lim_{n \to \infty} \frac{u_n+1}{u_n} = \lim_{n \to \infty} \frac{n+1}{n+3}$$

$$= \frac{\frac{n}{n} + \frac{1}{n}}{n/n + 3/n} = \frac{1 + 1/n}{1 + 3/n} = \frac{1+0}{1+0} = 1/1 = 1$$

since $\lim_{n \to \infty} \frac{u_n + 1}{u_n}$

The series is inconclusive.

Q23

$$\left[1/1^r + 1/2^r + 1/3^r + 1/4^r + \dots 1/n^r \right] = \sum_{n=1}^{\infty} 1/n^p$$

$$\rightarrow \left[2/1^2 + 2/2^2 + 2/3^2 + 2/4^2 + \dots 2/n^2 \right] = \sum_{n=1}^{\infty} 2/n^2$$

$\therefore p = 2$

thus $p > 1$, series will converge.

Q3

$$u_n = \frac{x^n}{(2n+1)^3}, \quad u_n + 1 = \frac{x^{n+1}}{(2n+2)^3}$$

$$\lim \frac{u_n + 1}{u_n} = \frac{x^{n+1}}{(2n+2)^3} \times \frac{(2n+1)^3}{x^n}$$

$$x \frac{(2n+1)^3}{(2n+2)^3} = \frac{8n^3 + 12n^2 + n + 1}{8n^3 + 24n^2 + 24n + 8}$$

DBT by n^3

thus $\dfrac{(8 + 2/n + 1/n^2 + 1/n^3)}{(8 + 12/n + 24/n^2 + 8/n^3)}$

as $n \longrightarrow \infty$

$1/n \longrightarrow 0$

$\dfrac{8x}{8} \cancel{\geq} x^{-1}$

$x < 1$

Q4

$\lim_{x \to 0} \left[\dfrac{\sin x - \cos x}{x^3} \right]$

using L'hopital's rule

$y = \left[\dfrac{\sin x - \cos x}{x^3} \right]$

$\dfrac{dy}{dx} = \left[\dfrac{\cos x + \sin x}{3x^2} \right]$

$\dfrac{d^2y}{dx^2} = \dfrac{-\sin x + \cos x}{6x}$

$\dfrac{d^3y}{dx^3} = \dfrac{-\cos x - \sin x}{6}$

$$\lim = \frac{-\cos 0 - \cos 0}{6} = \frac{-1-0}{6} = \frac{-1}{6}$$

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right] = -1/6$$