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DEPT: MECHATRONICS

COURSE: ENG 281

ASSIGNMENT 1 solution

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Using Auxillary equation $= m^2 - m - 2$

$\therefore$ $Gs = Cf + PI$

Cf - solve LHS = 0 $\therefore m^2 - m - 2 = 0$

$\therefore (m^2 - 2m + m - 2) = 0$

$m(m-2) + 1(m-2) \therefore = (m-2)(m+1)$

$\therefore m - 2 = 0 \qquad m + 1 = 0$

$m = 2 \qquad m = -1$

$\therefore$ $y = Ae^{-x} + Be^{2x}$

$PI = f(x) = 8$, Assume $y = c$

$\therefore \frac{dy}{dx} = 0$; $\frac{d^2y}{dx^2} = 0$

Substituting dy/dx and d^2y/dx^2 in the equation

$0 - 0 - 2c = 8$

$\frac{-2c}{-2} = \frac{8}{-2}$

$c = -4$

$Gs = Cf + PI$

$\therefore$ $y = Ae^{-x} + Be^{2x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{5x}$

$Gs = Cf + PI$

Cf = solve LHS $\therefore$ Auxiliary equation $= m^2 - 4 = 0$

$m^2 = 4 \quad \therefore \quad m = \sqrt{4} = m = \pm 2$

$\therefore$ $y = A\cosh 2x + B\sinh 2x$

PI = $f(x) = 10e^{5x}$: Assume $y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{5x}$; $\frac{d^2y}{dx^2} = 9Ce^{3x}$

Substitute d^2y/dx^2 and y in the given equation

$$9Ce^{5x} - 4Ce^{5x} = 10e^{5x}$$

$$5Ce^{5x} = 10e^{5x}$$

$$5C = 10$$

$$C = 2$$

$$\therefore y = 2e^{5x}$$

$Gs = Cf + PI$

$$y = A\cosh 2x + B\sinh 2x + 2e^{5x}$$

3) $d^2y/dx^2 + 2\,dy/dx + y = e^{-2x}$

$Gs = Cf + PI$

Cf = Solve $LHS = 0$ Auxiliary equation $m^2 + 2m + 1 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)}}{2 \times 1} = \frac{-2 \pm 0}{2}$$

$$m = -1 \text{ (twice)}$$

$$y = e^{-x}(A + Bx)$$

$PI \Rightarrow f(x) = e^{-2x}$ $\therefore$ Assume $y = Ce^{-2x}$

$$\frac{dy}{dx} = -2Ce^{-2x} \; ; \; \frac{d^2y}{dx^2} = 4Ce^{-2x}$$

Substitute the value of d^2y/dx^2 and dy/dx

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$Ce^{-2x} = e^{-2x}$$

$$C = 1 \quad \therefore y = e^{-2x}$$

$Gs = Cf + PI$

$$y = e^{-x}(A + Bx) + e^{-2x}$$

4) $\dfrac{d^2y}{dx^2} + 25y = 5x^2 + x$

$Gs = Cf + PI$

Cf = Solve $LHS = 0$ Aux equation $m^2 + 25 = 0$

$$m^2 = -25$$

$$m = \pm j5$$

$$\therefore y = A\cos 5x + B\sin 5x$$

$PI \div f(x) = 5x^2 + x$

Assume $y = (Cx^2 + Dx + E)$

$$\frac{dy}{dx} = 2(Cx + D)$$

$$\frac{d^2y}{dx^2} = 2C$$

Substitute d^2y/dx^2 and y in the equation.

$$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$2C + 25(x^2 + 25Dx + 25t) = 5x^2 + x$$

x^2 ! $25C = 5$

$C = 1/5$

x ! $25D = 1$

$$D = \frac{1}{25}$$

x^0 ! $2C + 25E = 0$

$2(1/5) + 25E = 0$

$25E = -2(1/5)$

$E = -2/125$

$\therefore y = (1/5 \times x^2) + (1/25 \times x) + (-2/125)$

$$y = \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$$

$$y = \frac{25x^2 + 5x - 2}{125}$$

$GS = CF + PI$

$$GS = A\cos 5x + B\sin 5x + \frac{25x^2 + 5x - 2}{125}$$

5) $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

$GS = CF + PI$

CF = Solve LHS = 0 Aux. equation $m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0 \Rightarrow m(m-1) - 1(m-1) = 0$

$m = -1$ (twice)

$y = e^{-x}(A + Bx)$

$PI \Rightarrow f(x) = 4\sin x$ $\therefore$ Assume $y = (\cos x + 1)\sin x$

$$\frac{dy}{dx} = -C\sin x + D\cos x$$

$$\frac{d^2y}{dx^2} = -(\cos x - 1)\sin x$$

Substituent general equation

$-(\cos x C) - 1) \sin x - 2(-C \sin x + D\cos x) + (\cos x + D \sin x = 4 \sin x$

$-C\cos x + D \sin x + 2x \sin x + 2D \cos x +$

$C(\cos x + 1) \sin x = 4 \sin x$

$(-D + 2C + D) \sin x + (-C - 2D + C) \cos x = 4 \sin x$

$\sin x : -D + 2C + D = 4$

$$\frac{2C}{2} = \frac{4}{2}$$

$$C = 2$$

$\cos x \quad -C - 2D + C = 0$

$-2 + 2D + 2 = 0$

$-2D = 0$

$D = 0$

$y = 2\cos x + 0 \sin x$

$y = 2 \cos x$

$GS = CF + PI \quad \therefore \quad GS = e^{-x}(A + Bx) + 2\cos x$

6) $d^2y/dx^2 + 4\, dy/dx + 5y = 2e^{-2x}$

CF => solve LHS = 0 $\qquad m^2 + 4m + 5 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4aC}}{2a} \qquad \frac{-4 \pm \sqrt{4^2 - 4(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm j$$

$y = e^{-2x}(A\cos x + B \sin x)$

$PI = f(x) - 2e^{-2x} \quad \therefore \quad y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x} \quad ; \quad \frac{d^2y}{dx^2} = 4Ce^{-2x}$

Substitute in the equation

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4C^{2-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$e^{-2x} : 4C - 8C + 5C = 2$

$C = 2$

$y = 2e^{-2x}$

$(GS) \; y = e^{-2x}(A\cos x + B \sin x) + 2e^{-2x}$

$x = 0 \quad ; \quad y = 1$

$1 = A + 2$

6) $A = -1$

$y = e^{-2x}(-\cos x + B\sin x) + 2e^{-2x}$

$\frac{dy}{dx} = e^{-2x}(\sin x + B\cos x) - 2e^{-2x}(-\cos x + B\sin x) - 4e^{-2x}$

If $x = 0$ and $\frac{dy}{dx} = -2$

$-2 = B + 2 - 4$

$B = 0$

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

$y = e^{-2x}\cos + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

7) $3\,d^3y/dx^2 - 2\,dy/dx - y = 2x - 5$

Cf $=$ LHS $= 0$: Aux equation $= 3m^2 - 2m + 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $\frac{-2 \pm \sqrt{(-2)^2 - 4(-3)}}{2\times1}$

$= \frac{2 \pm \sqrt{4 + 12}}{2}$ $= \frac{2 \pm 4}{2}$

$m = 3$ or -1

$\therefore y = Ae^{-3x} + Be^{3x}$

P_1 : $f(x) = 2x - 5$ But $y = (x + 1)$

$\frac{dy}{dx} = C$; $\frac{d^2y}{dx^2} = 0$

Substitute into the given equation

$3(0) - 2(c) - (cx + D) = 2x - 3$

$-2C - (x - D) = 2x - 3$

x : $-c = 2$

$c = -2$

x : $-2C - D = -3$

$-2(-c) - D = -3$

$4 - D = -3$

$D = 7$

$\therefore y = -2x + 7$

$G_s = C_f + P_I$

$G_s = Ae^{-x} + Be^{3x} + 2x + 7$

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$ ———— aux equation

$C_f \Rightarrow LHS = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4) = 0$

$m = 4$ or 2

$y = Ae^{2x} + Be^{4x}$

$P_I = f(x) = 8e^{2x}$ But $y = Ce^{4x}$

$\frac{dy}{dx} = 4Ce^{4x}$; $\frac{d^2y}{dx^2} = 16Ce^{4x}$

Substitute $\frac{d^2y}{dx^2}$ into general equation

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

e^{4x} : $16C - 24C + 8C = 8$.