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1(a) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(\sin(\cos x))}{x - \pi/2} \right]$

Using product rule to solve the numerator

$\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

for $u = x^2 - \pi/4$, $du/dx = 2x$

$v = \sin(\cos x)$, let $w = \cos x$ and $v = \sin w$

$\frac{dw}{dx} = -\sin x$, $\frac{dv}{dw} = \cos w$

$\frac{dv}{dx} = \frac{dw}{dx} \times \frac{dv}{dw} = -\sin x \times \cos w$

$= -\sin x(\cos(\cos x))$

$\frac{dy}{dx} = (x^2 - \pi/4) \cdot -\cos(\cos x)\sin x + \sin(\cos x) 2x$

For the denominator, $x - \pi/2$

let $y = x - \pi/2$

$\frac{dy}{dx} = 1$

$\lim_{x \to \pi/2} = \frac{(x^2 - \pi/4)(-\cos(\cos x)\sin x + \sin(\cos x) 2x}{1}$

$= \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right)\left(-\cos(\cos \pi/2)\sin \pi/2\right) + \sin(\cos \pi/2)^2(\pi/2)$

$\frac{\pi^2 - \pi}{4} (-1) + 0$

$= \frac{-\pi^2 + \pi}{4}$

$\therefore \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(\sin(\cos x))}{x - \pi/2} \right] = \frac{\pi(-\pi + 1)}{4}$

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(b) $\lim_{x \to \pi/2} \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right]$

$= \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right]$

$\lim_{x \to \pi/2}\left[\frac{3(\pi/2)^2+2(\pi/2)-1}{(\pi/2)+1}\right]$

$\dfrac{\frac{3\pi^2}{4}+\pi-1}{\frac{\pi+2}{2}}$

$\frac{3\pi^2}{4}+\frac{\pi}{1}-\frac{1}{1}$

$\frac{3\pi^2+4\pi-4}{4} = \frac{3\pi^2+4\pi-4}{4_2} \times \frac{\cancel{2}}{\pi+2}$

$= \frac{3\pi^2+4\pi-4}{2(\pi+2)}$

$\frac{3\pi^2-2(\cancel{\pi+2})}{2(\cancel{\pi+2})} = \frac{3\pi-2}{2}$

$\lim_{x \to \pi/2} \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right] = \frac{3\pi-2}{2} = \frac{3\pi}{2}-\frac{2}{2}$

$= \frac{3\pi}{2}-1$

c) $\lim_{x \to 2+\sqrt{3}} \cos\left(\sin^{-1}\frac{(x-2)}{(x-\sqrt{3}}\right)$

$\cos\left(\sin^{-1}\frac{(\cancel{2}+\sqrt{3}-\cancel{2}}{2+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}}\right)$

$\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)$

$\cos 60$

$\cos 60 = 1/2$

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Determine whether each of the following series is convergent

2(a) $\frac{2}{2\times3}+\frac{2}{3\times4}+\frac{2}{4\times5}+\frac{2}{5\times6}+\dots$

$U_n=\frac{2}{(n+1)(n+2)}$, $U_{n+1}=\frac{2}{(n+1+1)(n+1+2)}=\frac{2}{(n+2)(n+3)}$

Ratio $\frac{U_{n+1}}{U_n}=\frac{\cancel{2}}{(\cancel{n+2})(n+3)}\times\frac{(n+1)(\cancel{n+2})}{\cancel{2}}=\frac{n+1}{n+3}$

$\lim_{n\to\infty}\frac{\frac{n}{n}+\frac{1}{n}}{\frac{n}{n}+\frac{3}{n}}=\frac{1+\frac{1}{n}}{1+\frac{3}{n}}=\frac{1+0}{1+0}=1$ (inconclusive)

$\lim_{n\to\infty}U_n=\frac{2}{(n+1)(n+2)}=\frac{2}{n^2+3n+2}=\frac{\frac{2}{n^2}}{\frac{n^2}{n^2}+\frac{3n}{n^2}+\frac{2}{n^2}}=\frac{0}{1}=0$

$U_n=0$ ∴ it is convergent

b) $\frac{2}{1^2}+\frac{2}{2^2}+\frac{2}{3^2}+\frac{2}{4^2}+\dots$

Using Comparison test

$\frac{2}{1^2}+\frac{2}{2^2}+\frac{2}{3^2}+\frac{2}{4^2}+\dots=\frac{2}{1^p}+\frac{2}{2^p}+\frac{2}{3^p}+\frac{2}{4^p}+\dots$

$P=2$

when $P>1$, the series converge

Therefore the series is convergent.

(c) $U_n=\frac{1+2n^2}{1+n^2}$

If $\lim_{n\to\infty}U_n=0$, series is convergent

$U_n=$ ~~1+2(n~~ $\left[\frac{\frac{1}{n^2}+\frac{2n^2}{n^2}}{\frac{1}{n^2}+\frac{n^2}{n^2}}\right]$

$\lim n\to\infty$

$=\frac{0+2}{0+1}=2$

Since $U_n\neq0$, the series is Divergent

∴ The series is divergent

3. Find the range of values of x for which the series are absolutely convergent

$U_n=\frac{x^n}{(2n+1)^3}$, $U_{n+1}=\frac{x^{n+1}}{(2n+1)+1)^3}$

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$$U_{n+1} = \frac{x^{n+1}}{(2n+2+1)^3}$$

x^{n+1} Ratio: $\frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$

$$\frac{x^n + x^1}{(2n+2)^3} \times \frac{(2n+1)^3}{x^n} = \frac{x^1(2n+1)^3}{(2n+3)^3}$$

$$\frac{x(2n+1)(2n+1)(2n+1)}{(2n+3)(2n+3)(2n+3)}$$

$$= \frac{x(8n^3+12n^2+6n+1)}{8n^3+36n^2+54n+27}$$

$$x\frac{\left[\frac{8n^3}{n^3}+\frac{12n^2}{n^3}+\frac{6n}{n^3}+\frac{1}{n^3}\right]}{\frac{8n^3}{n^3}+\frac{36n^2}{n^3}+\frac{54n}{n^3}+\frac{27}{n^3}}$$

$$x\frac{\left[8+12/n+6/n^2+1/n^3\right]}{8+36/n+54/n^2+27/n^3}$$

$$\lim_{n\to\infty} = x\left[\frac{8+0+0+0}{8+0+0+0}\right] = \frac{8x}{8} = x$$

$$\lim_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right| = x$$

For absolute convergence $\lim_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right| < 1$

Series is convergent when $-1 \leq x \leq 1$

4. $\lim_{x\to 0}\left\{\frac{\sin x - \cos x}{x^3}\right\} = \lim_{x\to 0}\left\{\frac{\cos x + \sin x}{3x^2}\right\}$

$$\lim_{x\to 0}\left\{\frac{-\sin x + \cos x}{6x}\right\}$$

$$\lim_{x\to 0}\left\{\frac{-\cos x - \sin x}{6}\right\}$$

$$\left\{\frac{-\cos 0 - \sin 0}{6}\right\} = \left\{\frac{-1-0}{6}\right\}$$

$$= \frac{-1}{6}$$